Collision

$\downarrow$ thermalization

QGP

$\downarrow$ hadronization

Hadron gas

$\downarrow$ freezeout

detector

Idea: How does equilibrium $(\Delta Q)^2$ change with evolution?

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$$\frac{(\Delta Q)^2}{N_{ch}} \approx 0.25 \quad \text{in QGP}$$

$$\approx 0.7 \quad \text{in HG } (\pi\text{ions} + \text{resonances})$$

Q-conserved $\Rightarrow$ $\Delta Q \sim$ "frozen"

ΔQ changes only by diffusion

If $\tau_{diff} \gg \tau_{expansion}$ then ΔQ is "frozen"

τ_{diff} depends on the scale of fluctuation $(1/k)$.

Fluctuations occur on all scales.

$\tau_{diff} \sim \frac{1}{\gamma k^2}$, i.e. longer wavelength modes (small k) relax slower.

γ - diffusion coefficient; depends on time

$\tau_{diff} \sim \frac{1}{\gamma k^2}$

We need :

$\tau_{diff, QGP} \ll \tau_{exp}$ to have eq. QGP fluctuations

and

$\tau_{diff, HG} \gg \tau_{exp}$ to "freeze" those fluct. during HG phase

$\gamma \downarrow$ with time $\Rightarrow$ one can hope to achive this by tuning $k \sim 1/\Delta y$. Δy - rapidity window.

We assume that $\tau_{diff, QGP} \ll \tau_{exp}$

- What are the conditions that QGP fluctuation magnitude is preserved during HG stage?
- How much fluct. relaxes towards HG equilibrium value during HG stage?

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Need: equation for $(\Delta Q)^2(\tau)$.

$n(y,\tau)$ - density of Q

$f(y,\tau) = n(y,\tau) - n_0$: $(\Delta Q)^2 = \int_{\Delta y} f^2\, dy$

Assume Bjorken scenario ($n_0 = Q_{total}/y_{total}$)

Since f is a density of conserved quantity:

$\partial_\tau f = \gamma \partial_y^2 f$. (i) f - small → linear equation
(ii) $\Delta y \gg \delta y_{coll}$ (some idealization)

$\gamma = \gamma(\tau)$ - diffusion coefficient (simple estimate: $\gamma \approx \frac{(\delta y_{coll})^2}{2\tau_{free}}$)

Need noise:

$\partial_\tau f = \gamma \partial_y f + \xi$ Langevin eq.

Correlation function for ξ is determined from

$\langle f(y) f(y') \rangle_{equilibrium} = \chi(y-y')$

Fourier transform:

$\partial_\tau f_k = -\gamma k^2 f_k + \xi_k$, $\langle \xi_k(\tau) \xi_{k'}(\tau') \rangle = 2\gamma k^2 \delta_{kk'} \delta(\tau-\tau') \cdot \chi_k$

↓

Fokker-Plank equation for $P[f_k]$

Gaussian solution: $P[f_k] = \prod_k P_k(f_k)$; $P_k(f_k) \sim \exp\left(-\frac{f_k^2}{2\sigma_k^2(\tau)}\right)$

↓

$\partial_\tau \sigma_k^2 = -2\gamma k^2 (\sigma_k^2 - \chi_k)$ $\langle f_k^2 \rangle = \sigma_k^2$

↓

$\sigma_k^2(\tau) = \chi_k + [\sigma_k^2(0) - \chi_k] \exp\left(-2k^2 \int_0^\tau \gamma\, d\tau\right)$

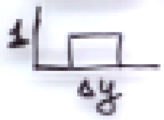
$$\sigma_k^2(\tau) = \chi_k + [\sigma_k^2(0) - \chi_k]\exp\left(-2k^2 \underbrace{\int_0^\tau \gamma\, d\tau}\right)$$

$$(\Delta y_{diff})^2 \equiv 2\int_0^\tau \gamma\, d\tau = \int_0^\tau (\delta y_{coll})^2 \underbrace{\frac{d\tau}{\tau_{free}}}_{\#\text{ of collisions}}$$

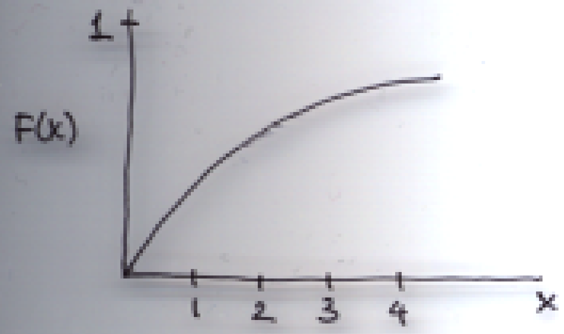
Finally:

$$\langle(\Delta Q)^2\rangle = \int_{\Delta y} \langle f^2\rangle\, dy = \int \frac{dk}{2\pi}|c_k|^2 \underset{\sigma_k^2(\tau)}{\underset{\parallel}{\langle f_k^2\rangle}}$$

Fourier of



$$= \langle(\Delta Q)^2\rangle_{equilibrium}\cdot F\left(\frac{\Delta y_{diff}}{\Delta y/2}\right) \qquad (\text{let } \langle(\Delta Q)^2\rangle_0 = 0)$$



$$F(x) = 1 + \frac{x}{\sqrt{\pi}}\left(1 - e^{-1/x^2}\right) - \operatorname{erf}\left(\frac{1}{x}\right)$$

For $\langle(\Delta Q)^2\rangle_0 \neq 0$:

$$\langle(\Delta Q)^2\rangle_\tau = \langle(\Delta Q)^2\rangle_{eq}\cdot F(x) + \langle(\Delta Q)^2\rangle_0 (1 - F(x))$$

with "time" $x \sim \Delta y_{diff}$

Estimates

$$\frac{\langle(\Delta Q)^2\rangle}{\langle(\Delta Q)^2\rangle_{HG}} = F(x) + \frac{\langle(\Delta Q)^2\rangle_{QGP}}{\langle(\Delta Q)^2\rangle_{HG}}\left(1 - F(x)\right)$$

$$x = \frac{\Delta y_{diff}}{\Delta y / 2}$$

Need Δy_{diff} : $\Delta y_{diff} \approx \delta y_{coll} \cdot \sqrt{N_{coll}}$

$\delta y^{\pi}_{coll} \approx 0.8$; SPS $N^{\pi}_{coll} \sim 5$ $\Delta y_{diff} \approx 1.8$

RHIC $N^{\pi}_{coll} \sim 8$ $\Delta y_{diff} \approx 2.2$

Assume $\frac{\langle(\Delta Q)^2\rangle_{QGP}}{\langle(\Delta Q)^2\rangle_{HG}} \approx \frac{1}{3}$

Then: SPS : $\frac{(\Delta Q)^2_{freezeout}}{(\Delta Q)^2_{HG}} \approx 0.8$ $(\Delta y = 2)$

RHIC : ≈ 0.7 $(\Delta y = 4)$

$(\Delta Q)^2$ near QCD critical point

$\langle(\Delta Q)^2\rangle_{HG} \sim 0.7$ of free pions, mostly due to $\rho^0, \omega \to \pi^+\pi^-$

Near QCD critical point:

σ is a resonance with mass $\to 0$.

$\sigma \to \pi^+\pi^-$ reduces $\dfrac{(\Delta Q)^2}{N_{ch}}$ (like ρ^0 or ω)

Crude estimate: $\dfrac{(\Delta Q)^2 \text{ with } \sigma}{(\Delta Q)^2 \text{ without } \sigma} \approx 0.8$

Contribution of σ is non-monotonous function of $\sqrt{s}$.

+ other signatures must appear and disappear (such as large Φ_{p_T}.)

* * *

Conclusions

- To see "frozen" charge fluctuation further back in the history of collision one needs wider Δy.
- For realistic Δy rescattering at HG stage rises $\langle (\delta Q)^2 \rangle$ towards HG eq. value (by factor $\sim 2 \div 3$).
- Charge fluctuations are smaller near QCD critical point.