Balance Functions: A Signal of Late-Stage Hadronization
Motivation

Suppose one could identify balancing charges? (e.g. $K^+, K^-$)

- Hadrons appear at $\tau \approx 0.5$ fm/c.
- String dynamics separate balancing $Q-\bar{Q}$ by $\Delta y \sim 1$.
- Strangeness annihilates with time, reduces probability of small $\Delta y$.

- Hadronization at 5-10 fm/c into collision, $T \approx 165$.
- Many $q\bar{q}$ pairs created during hadronization.
- Balancing charges separated by $\Delta y \sim v_{\text{therm}}$.

Narrow distribution in $\Delta y$ signals late production of $q\bar{q}$ pairs. → novel phase persisted substantial time.
Creation of $q\bar{q}$ Pairs at RHIC

During hadronization, $q\bar{q}$ pairs are created for three reasons.

1. Gluons $\rightarrow$ Hadrons.
   At fixed $T$, each gluon should make $\approx 1$ hadron due to entropy conservation.

2. Quarks $\rightarrow$ Hadrons.
   At fixed $T$, each quark should make $\approx$ one hadron due to entropy conservation.

   (e.g. DCC) Probably a small fraction of particle creation.

- Each hadron contains at least two quarks, so number of quarks should more than double during hadronization.
- Coalescing quark gas would require rise in $T$ to keep $\Delta S \geq 0$. 
What are Balance Functions?
Given the existence of a particle with momentum $p_1$, balance functions describe the probability of seeing a particle of opposite charge with momentum $p_2$.

$$B(p_2|p_1) \equiv \frac{1}{2} \{\rho(+Q,p_2|-Q,p_1) - \rho(-Q,p_2|-Q,p_1)$$
$$+ \rho(-Q,p_2|+Q,p_1) - \rho(+Q,p_2|+Q,p_1)\}$$

Here $\rho(b,p_2|a,p_1)$ is the conditional probability,

$$\rho(b,p_2|a,p_1) = \frac{N(a,p_1;b,p_2)}{N(a,p_1)}$$

Common binning choice:
1. $p_1$ is anywhere in detector.
2. $p_2$ refers to relative rapidity.

Can be applied to specific particle/antiparticle pairs, e.g. $\pi^+/\pi^-$, or to specific charges, e.g. (all antibaryons)/(all baryons).
Properties of Balance Functions

- Normalized to unity:
  If $+Q/-Q$ refers to ALL +/- particles
  \[
  \sum \hat{p}_2 \mathcal{B}(p_2|p_1) = 1
  \]

- Works for both cases:
  1. $\sum q_i = 0$, e.g. strange/antistrange
  2. $\sum q_i \neq 0$, e.g. baryon/antibaryon

- Normalization reduced for finite acceptance or for using subset of particles, e.g. analyze only $K^+/K^-$. 

- May be analyzed event-by-event.
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Statistical Error and Multiplicity $M$

$$\rho(b, p_2|a, p_1) = \frac{N(a, p_1; b, p_2)}{N(a, p_1)}$$

- Statistical error for numerator $\propto \sqrt{M^2}$.
- Denominator also increases $\propto M$.
- Error $\propto 1/\sqrt{N_{\text{events}}}$, independent of $M$.
- $p\bar{p}$, $K^+K^-$ and $\pi^+\pi^-$ give similar errors.
- $10^5$ events makes good balance function.
**Balance Functions from Jets**

- Similar analyses performed with:
  - ppdata: D. Drijard et al., NPB 155 (1979) 269.
    D. Drijard et al., NPB 166 (1980) 233.
    M. Althoff et al., ZPC 17 (1983) 5.
    P.D. Acton et al., PLB 305 (1993) 415.

- Several pairs analyzed, e.g. $\Lambda\bar{\Lambda}$.
- JETSET fits data.

Thanks to T. Sjöstrand for references!
Bjorken 1-d expansion:
Time: \( \tau = \sqrt{t^2 - z^2} \)
Position: \( \eta = \tanh^{-1}(z/t) \)
Collective velocity: \( y = \eta \).

- Pairs generated thermally at same \( \eta \) with same collective rapidity \( y \).
- \( B(\Delta y) \) determined by \( T/m \).
- Heavier particles provide greater sensitivity.
Diffusion: An Analytic Picture

Diffusion Eq:
\[
\frac{\partial}{\partial \tau} f(\tau, \eta) = -\frac{\beta}{\tau} \frac{\partial^2}{\partial \eta^2} f(\tau, \eta),
\]
\[
\beta = \frac{v_t}{(n\tau \sigma)}
\]

Solution:
\[
f(\tau, \eta) \sim \exp\left(-\frac{\eta^2}{2\sigma_\eta^2}\right),
\]
\[
\sigma_\eta^2 = 2\beta \ln(\tau/\tau_0)
\]

- No diffusion when
  1. \(\beta = 0\) (Coll. Rate\(\to\) \(\infty\))
  2. \(\tau = \tau_0\) (No Collisions)
- \(\sigma_\eta\) largest for small \(\tau_0\).

\[
\sigma_{\text{balance}}^2 = 2(\sigma_{y,\text{therm.}}^2 + \sigma_\eta^2)
\]
Collisions and Annihilations: A Simple Model

Procedure:
1. Generate pair thermally at $\eta = 0, \tau = \tau_0$.
2. Follow straight-line trajectories between collisions.
3. Perform $N_{\text{coll}}$ collisions randomly in $\ln \tau$.
4. Readjust momenta to local thermal conditions.
   $T = 225 - 7.5(\tau - 1), \tau_f = 15$

Annihilations:
- Modeled by convoluting pairs.
- If annihilation rate = creation rate $\rightarrow$ no effect.

Collisions/Annihilations magnify sensitivity to creation time!

Scott Pratt

NSCL/MSU
Collisions: Model Summary

If $\tau_0 \approx 1 \text{ fm/c}$,

$$N_{\text{coll.}} \sim 6$$

If $\tau_0 \approx 9 \text{ fm/c}$,

$$N_{\text{coll.}} \sim 2$$

Even pions become sensitive to hadronization time!
Far reaching implications

For example,

A. If measured balance functions have significant extra strength near $\Delta y=0$, characteristic of $T \sim 165$ MeV, then either
   - Large numbers of new charges were created late in the reaction, e.g. hadronization of gluons.
   - Mean free paths of partons were anomalously short during very early times.

B. If $pp \& AA$ balance functions appear identical,
   - Gluonic modes did not contribute to entropy for a substantial time.
   - Quarks and antiquarks did not contribute to entropy as separate particles (unless temperature jumped at hadronization).
   - Most explanations of strangeness enhancement are wrong.
   - Most jet energy loss calculations are misguided.
   - QGP explanations of $J/\Psi$ suppression are misguided.
Conclusions

- Provide clear signal of late stage hadronization — a long-lived QGP?

- Strangeness/Antibaryon production issues can be studied.
- Gating on $p_t$ allows one to study production as function of $r_\perp$.