The QCD Mixed Phase: Shaken but not stirred

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1st Order Phase Transition in Ion Collisions?

- familiar flow signals probe small bubbles
- fluctuation signals probe large bubbles

I. Extraordinary fluctuations from 1st order transition

- simple example: baryon fluctuations at high density

II. Diffusion dynamics for conserved order parameter

- slow rapidity diffusion → fluctuations can survive

with D. Bower
& C. Pruneau

Baryon Density: the Other Order Parameter

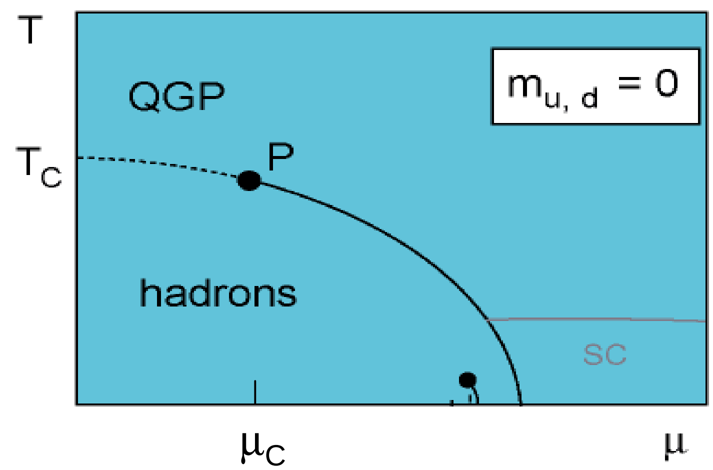
QCD with two massless quarks

$\mu = 0$: 2nd order chiral transition

$\mu = \mu_c$: tricritical point

$\mu > \mu_c$: 1st order

μ = baryon chemical potential



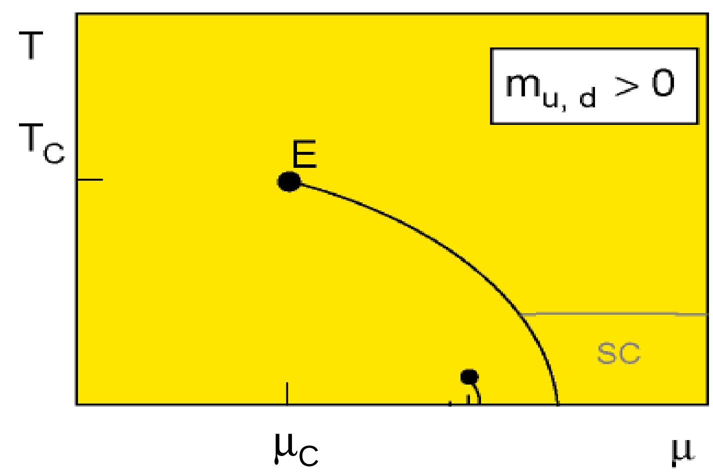
Berges, Rajagopal; Stephanov et al; Shuryak et al.

Real World QCD

$\mu = 0$: "smooth" crossover

$\mu = \mu_c$: critical point

$\mu > \mu_c$: 1st order transition ~ liquid-gas



1^{st} Order Transition ⇒ Fluctuations

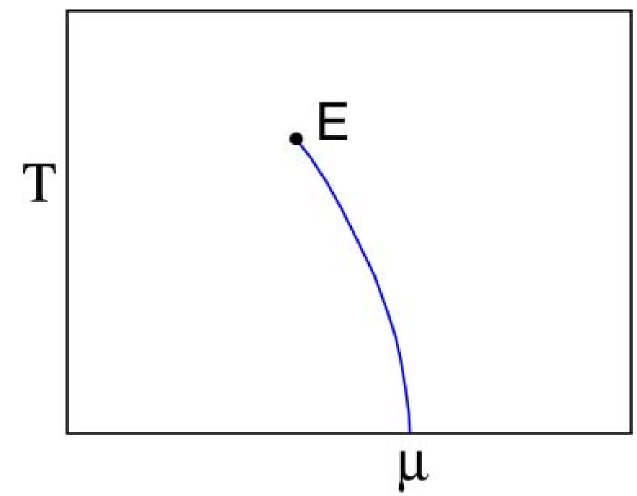
net baryon density > 0
Berges & Rajagopal, et al.

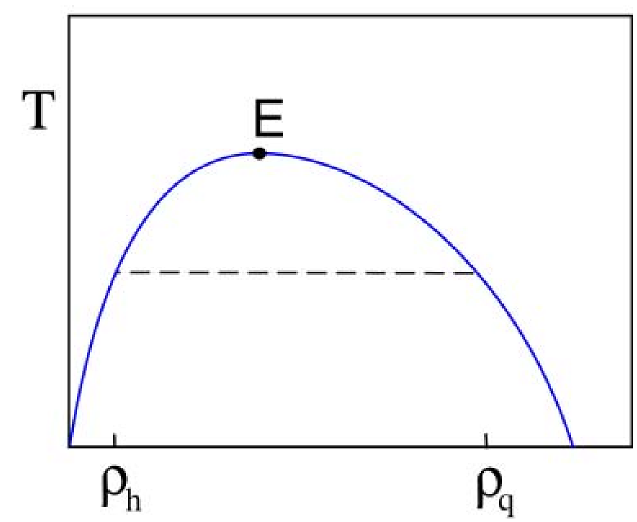
1^{st} order ⇒ phase separation

- **bimodal distribution** of densities -- "mixed phase"
- hadronic bubbles, density ρ_h plasma, ρ_q
- $\rho = 0$ transition may be "nearly" 1^{st} order

signatures of bubbles

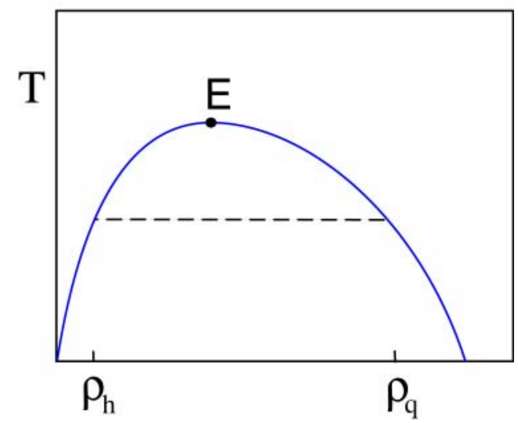
- interferometry Pratt, Siemens
- variance of net baryon number





Baryon Number Fluctuations

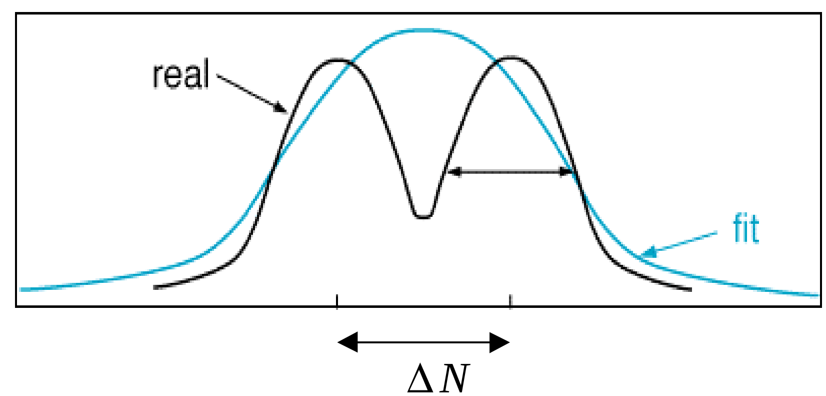
bimodal distribution of densities within each event "mixed phase"



- plasma fraction f
- N net baryons in Δy

variance:

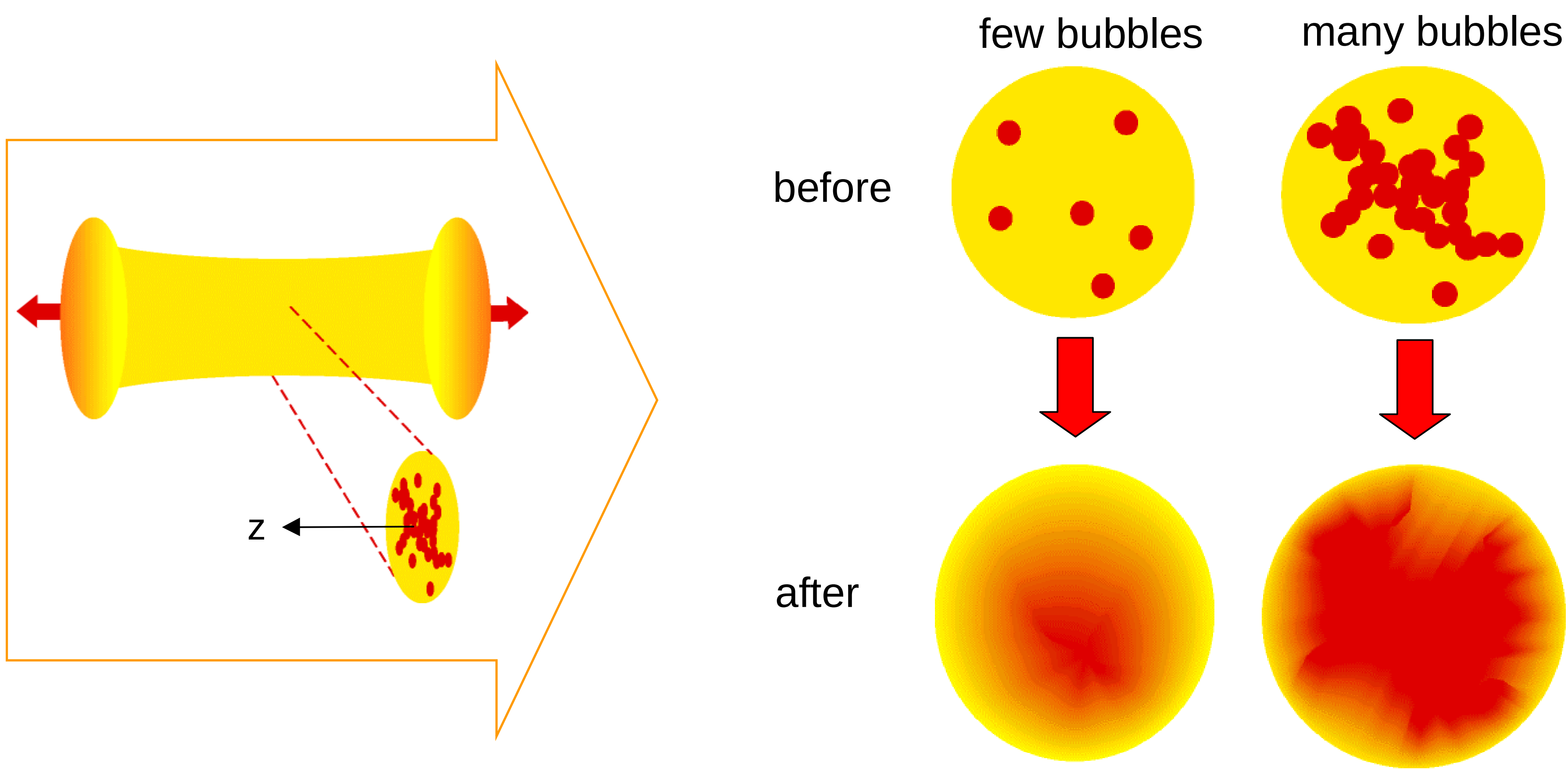
$$V \equiv \langle N^2 \rangle - \langle N \rangle^2$$
$$= f(1-f)(\Delta N)^2 + \ldots$$



ask: do these fluctuations survive?

Bubble Formation ⇒ Baryon Fluctuations

phase transition ⇒ baryonic bubbles
number of bubbles varies event by event



find: baryon number **fluctuates** in a rapidity slice

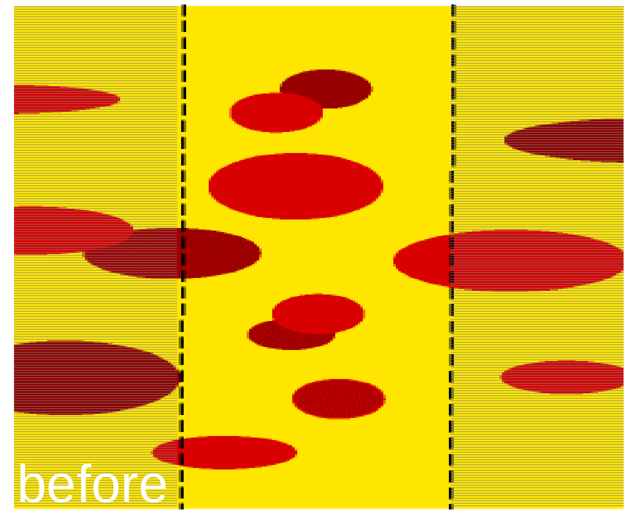
Rapidity Fluctuations Can Survive

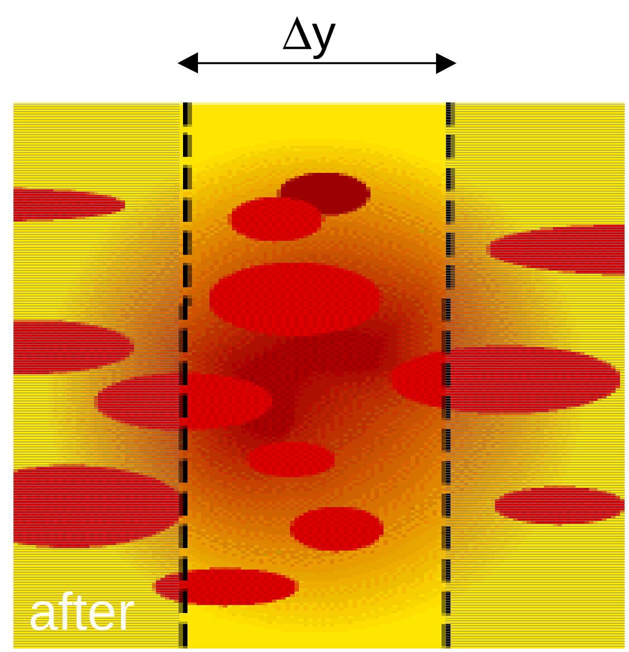
expansion **fast:** $\delta z \sim \tau$

diffusion **slower:** $\delta z \sim (D\tau)^{1/2}$

diffusion const. $D \sim$ 2-10 fm

Prakash et al.; Heiselberg et al.





find: expansion inhibits spread in rapidity – **observable!**

S. Gavin, nucl-th/9908070

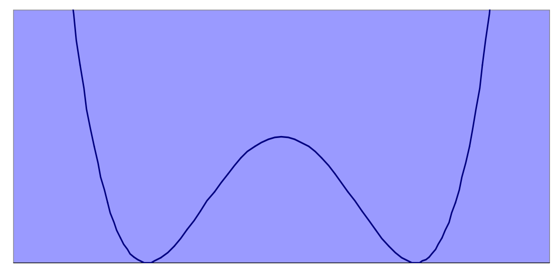
Phase Separation

A.J. Bray, Ad. Phys. 43 (1994) 357; M. Grant et al.

free energy vs. baryon density ρ for mixed phase:

$$f = \kappa (\nabla\rho)^2 + f_0(\rho) \quad \text{where} \quad f_0 = -m^2 (\rho - \rho_c)^2 + \lambda (\rho - \rho_c)^4$$

- two phases $\rightarrow \rho_h, \rho_q$
- Ising-like near T_C
- surface tension $\sigma \propto \kappa^{1/2}$



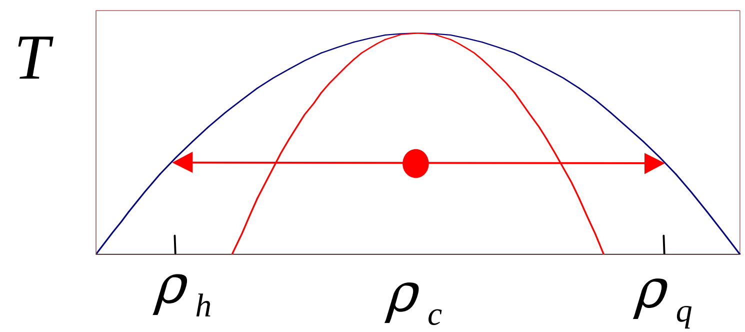
diffusion dynamics for conserved ρ

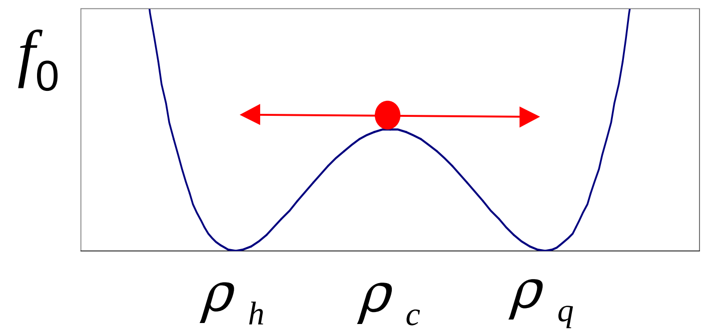
$$\partial\rho/\partial t = M\nabla^2 \mu \quad \text{where} \quad \mu = f_0' - \kappa\nabla^2\rho$$

- diffusion coefficient $D = 2m^2M$

Phase Separation Dynamics

quench system deep into coexistence region





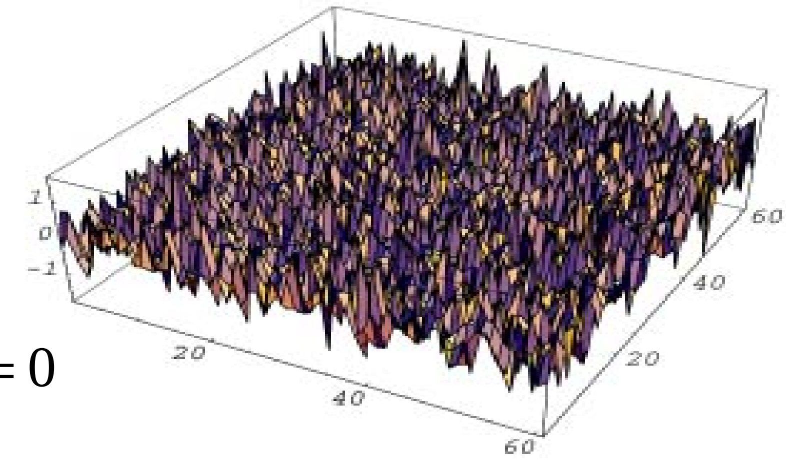
dynamics: $$\partial\rho/\partial t = M\nabla^2\left(f_0' - \kappa\nabla^2\rho\right)$$

bubbles from runaway density waves

- fastest growing mode – time scale $\tau_R \propto \lambda_R^2/D \rightarrow 1$ fm
- wavelength $\lambda_R \propto$ corr. length $\rightarrow 1$ fm
- nonlinear evolution for $t >> \tau_R$

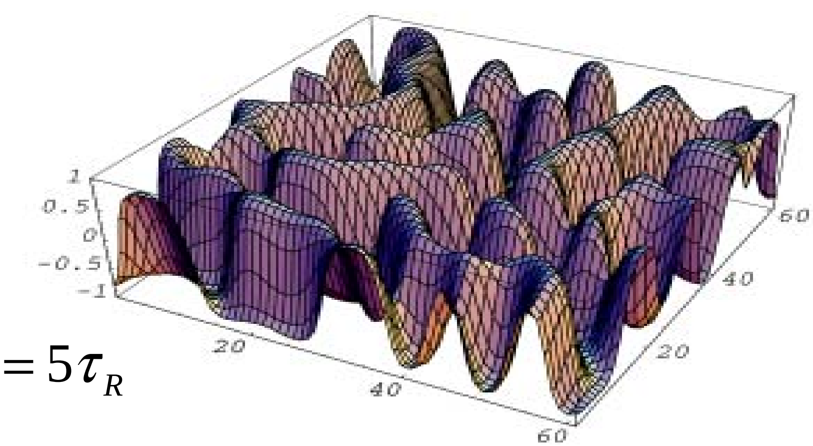
Benchmark Simulations

random initial distribution (2+1d)



$t = 0$

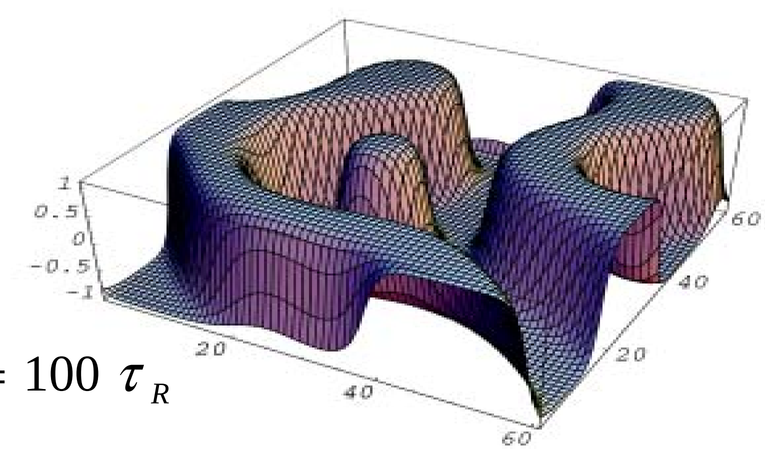
early domain formation



$t = 5\tau_R$

phase separation

- no expansion
- eternal 50-50% mixed phase
- droplets merge, reduce surfaces



$t = 100\ \tau_R$

Phase Separation in Ion Collisions

S. Gavin, nucl-th/9908070; D. Bower and S. Gavin, *in progress*

longitudinal flow + diffusion:

$$\frac{\partial\rho}{\partial\tau}+\frac{\rho}{\tau}=M\nabla^2\left(f_0'-\kappa\,\nabla^2\rho\right)$$

- $\partial_\mu(\rho v^\mu + j_D{}^\mu) = 0$; diffusion current j_D
- Bjorken flow: $v_z \sim z/\tau$, proper time τ

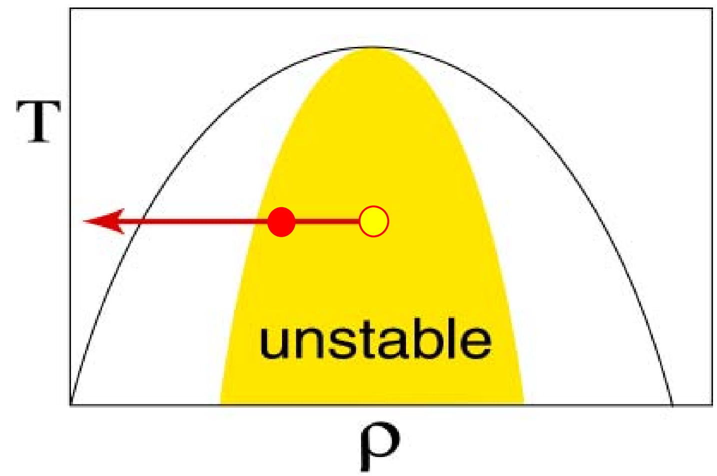
evolution following a quench

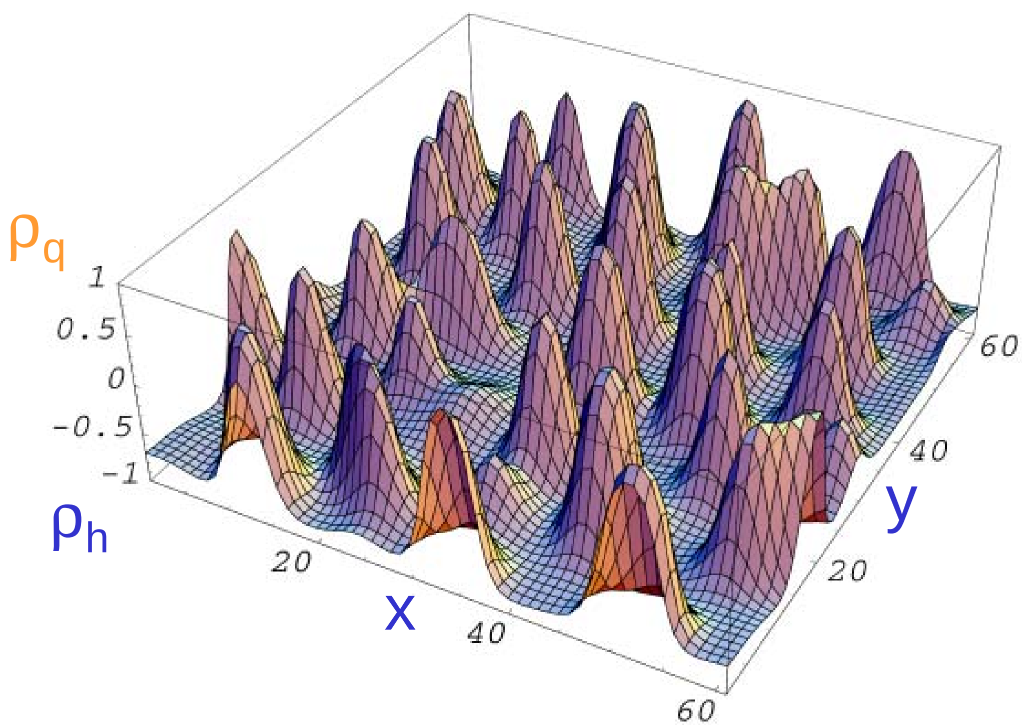
- quench at $1 < \tau_0 < 10$ fm
- assume $\tau_R \sim$ correlation length ~ 1 fm
- outcome depends on τ_0 / τ_R

Evolution with Expansion

expansion $\Rightarrow \langle\rho\rangle \propto \tau^{-1}$

stable for $\tau > 2.3\,\tau_0$





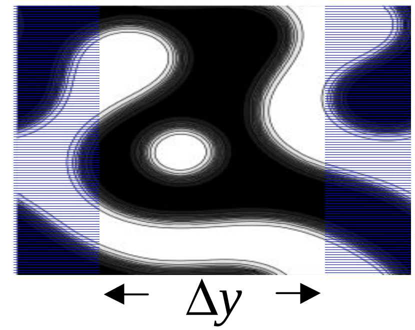
initial $\tau_0 = 5$ fm

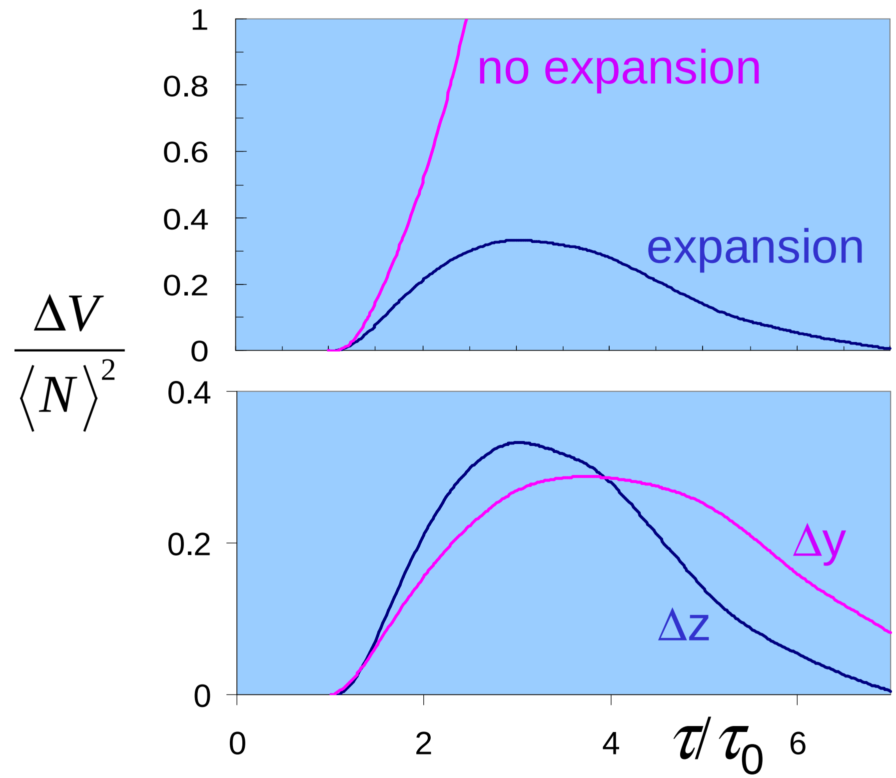
$\tau \sim 2\tau_0 = 10\tau_R \sim 10$ fm

find: droplets, but few connected regions

Variance of Baryon Number

- simulate 1000 "events" – initial τ_0= 5 fm
- baryon number N for strips: $\Delta y \sim \Delta z/\tau$ or Δz
- compute $V = \langle N^2\rangle - \langle N\rangle^2$, $\Delta V \equiv$ excess over Poisson





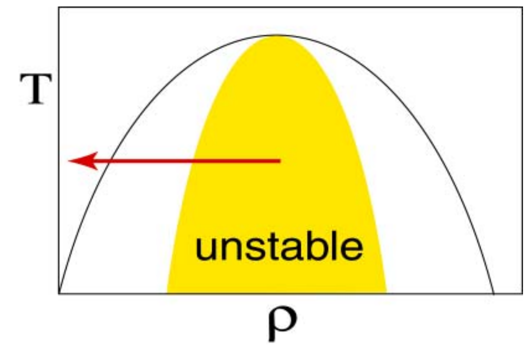
Find:

- variance rises initially
- falls after stabilized ($\tau = 2.3\tau_0$)
- Δy falls less quickly

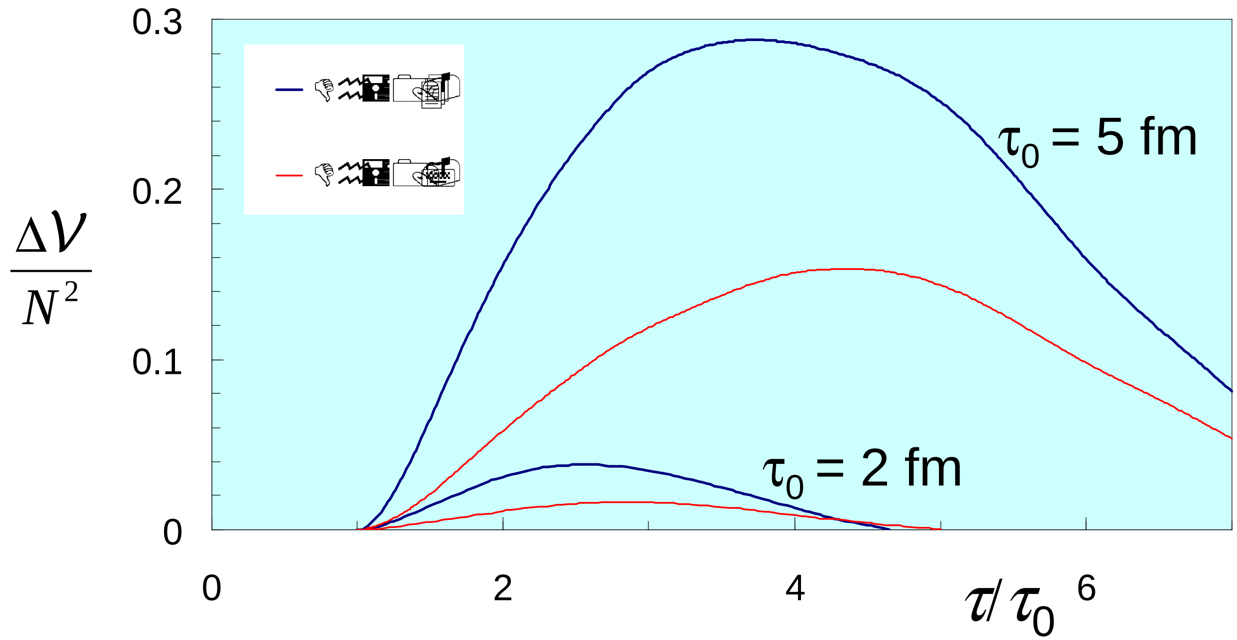
Variance in Collisions?

need $\tau_0/\tau_R >> 1$ for large ΔV

- expansion time scale ~ τ_0
- phase separation time ~ τ_R



droplets must form before system exits unstable region



Summary: Baryon Fluctuations

"Ordinary" fluctuations

- QGP vs. HG — Jeon & Koch; Asakawa et al. Stephanov and Shuryak

Extraordinary? – 1st order transition

- meson fluctuations — Heiselberg, Baym, Jackson
- baryon fluctuations — S.G.

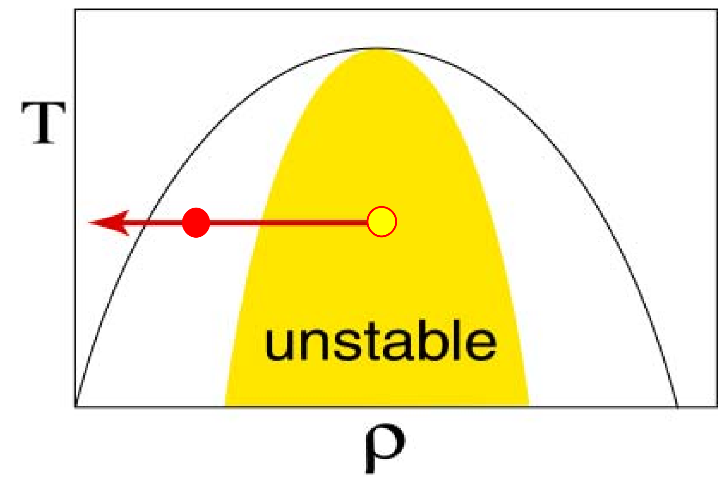
Baryon fluctuations at high density – theory easier

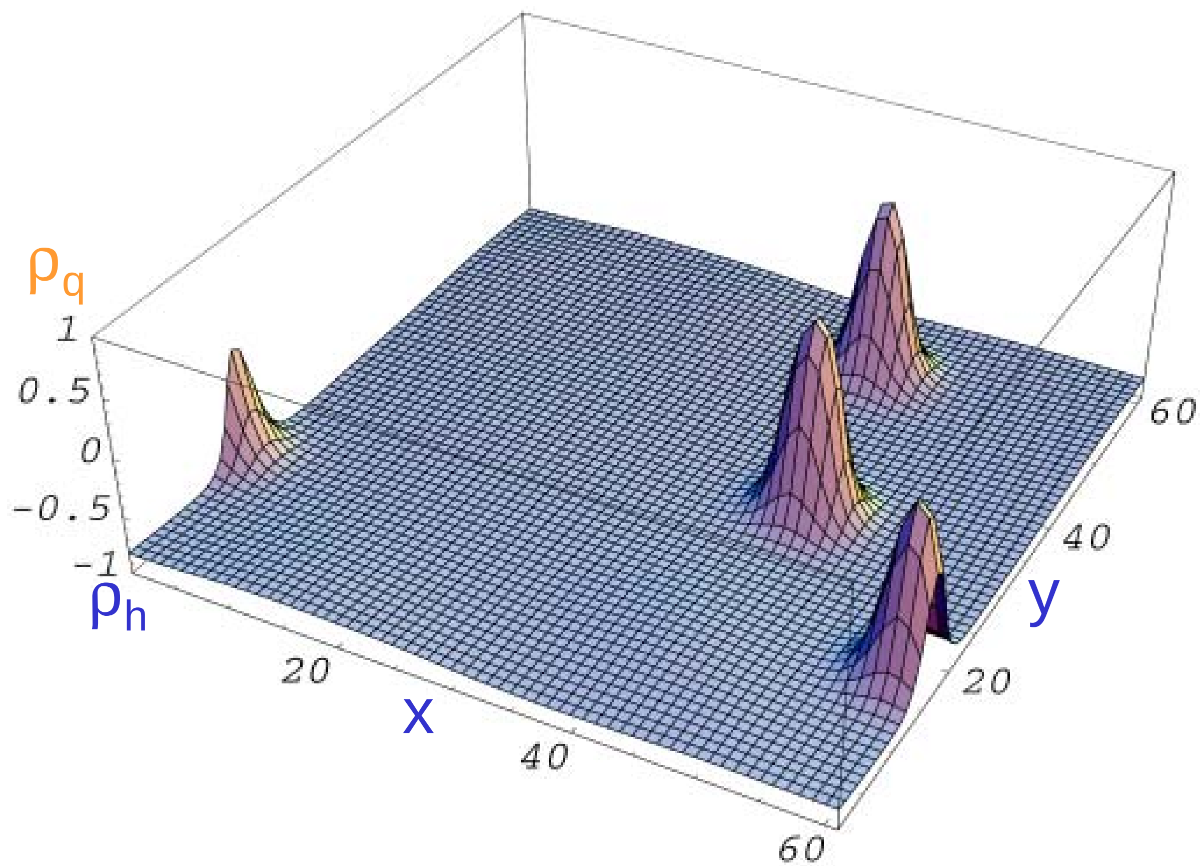
- conserved – fluctuations in rapidity bins preserved
- time to develop? **Maybe** — Bower, S.G.

Latest Evolution

expansion stabilizes system:

- stable for $\tau > 2.3\tau_c$
- diffusion smears droplets





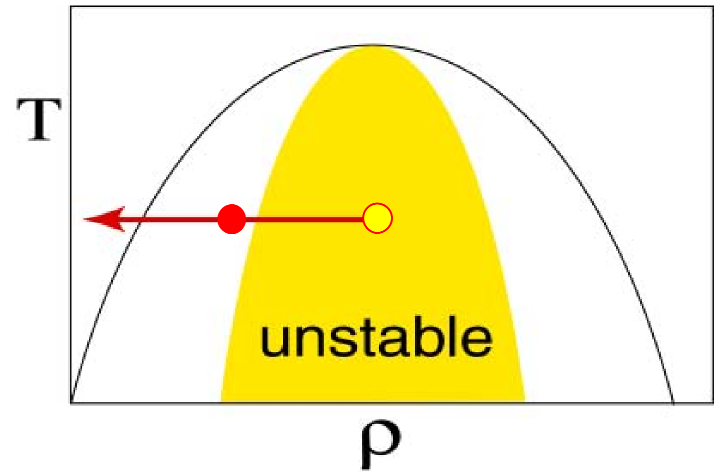
expansion: droplets distributed over a larger volume

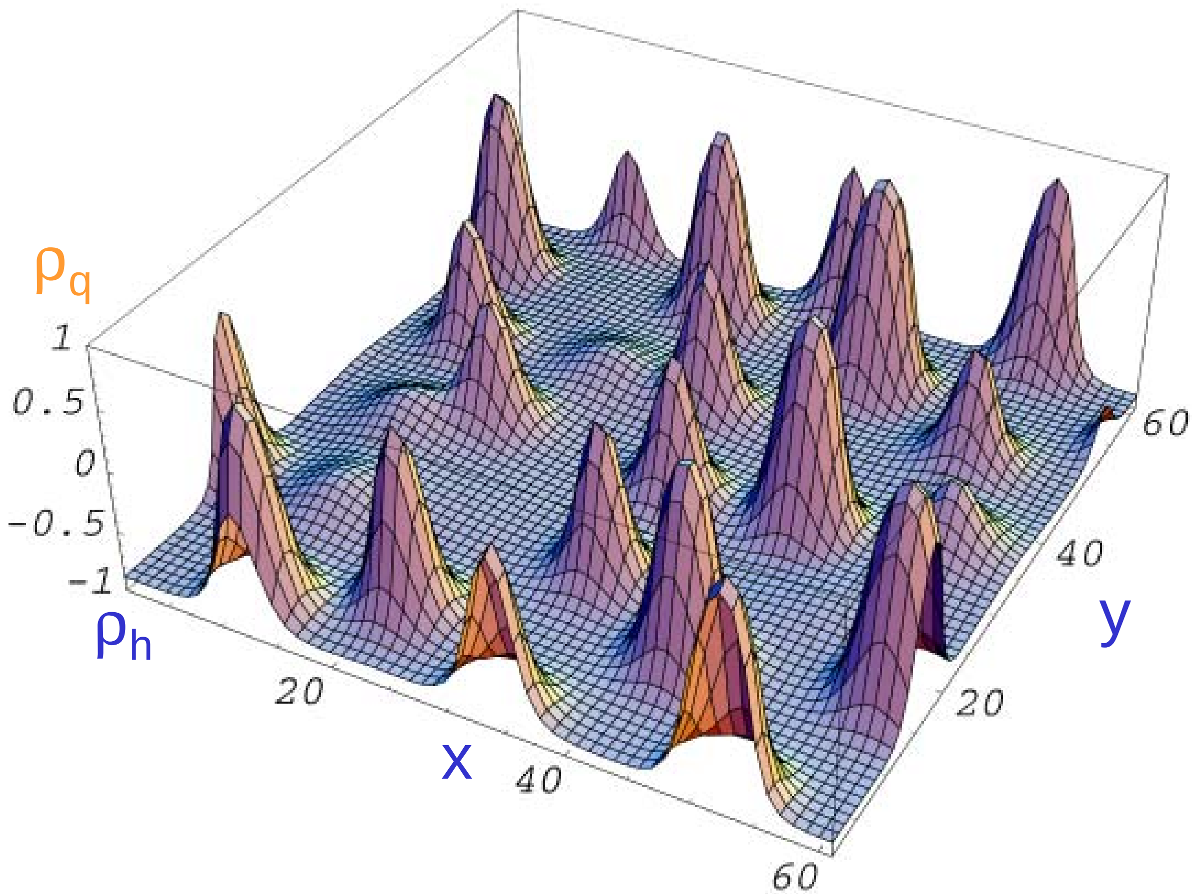
$\tau \sim 5\tau_c = 25\tau_{sp}$

Later Evolution

expansion stabilizes system:

- stable for $\tau > 2.3\tau_c$





expansion spreads droplets over a larger volume

$\tau \sim 3\tau_c = 15\tau_{sp}$

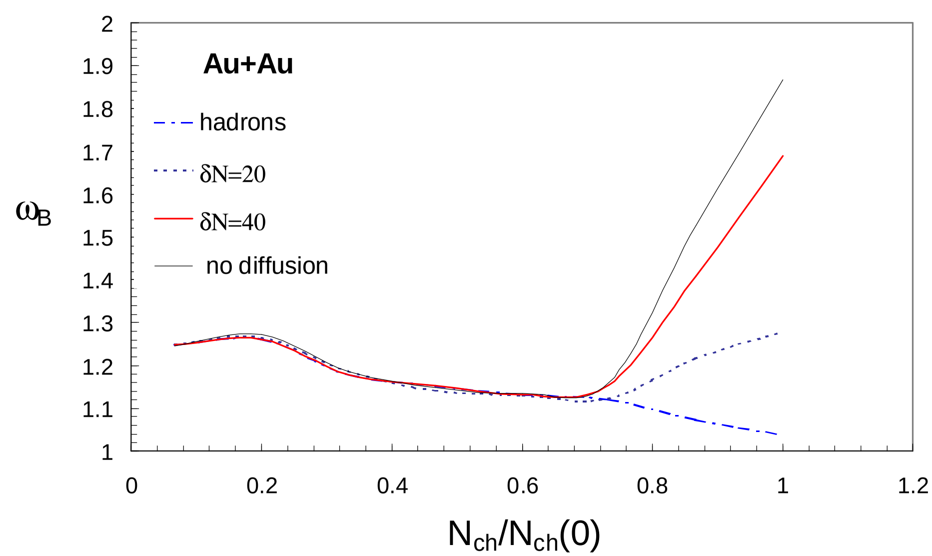
Observable Fluctuations

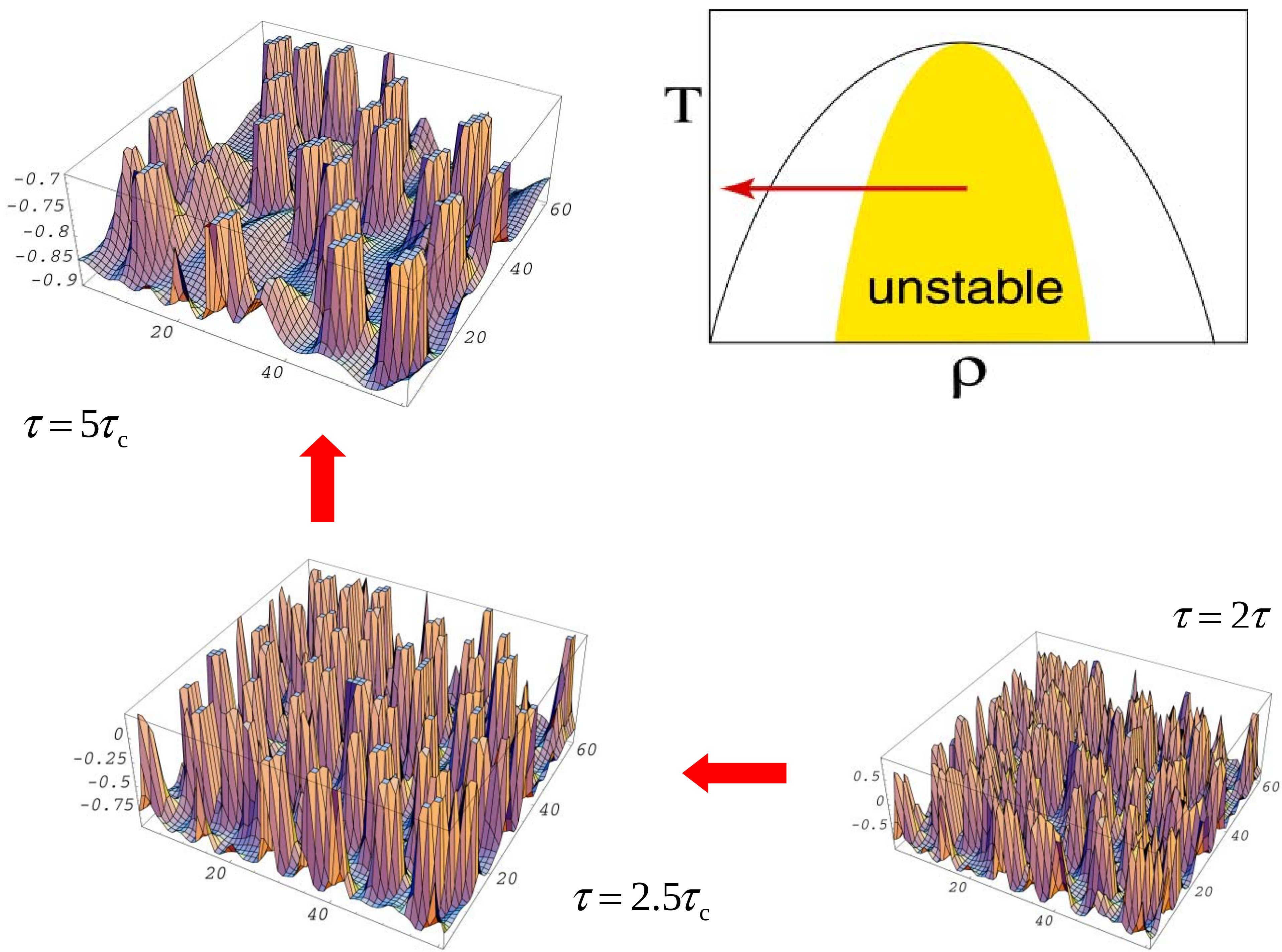
high density phase:

- fraction $f \sim 0.25\,\{1 - [b/(3\text{ fm})]^2\}$, diluted by diffusion
- density contrast $\delta N = N_q - N_h$
- event generator

compute: $\omega_B = V_B / (N + \overline{N})$

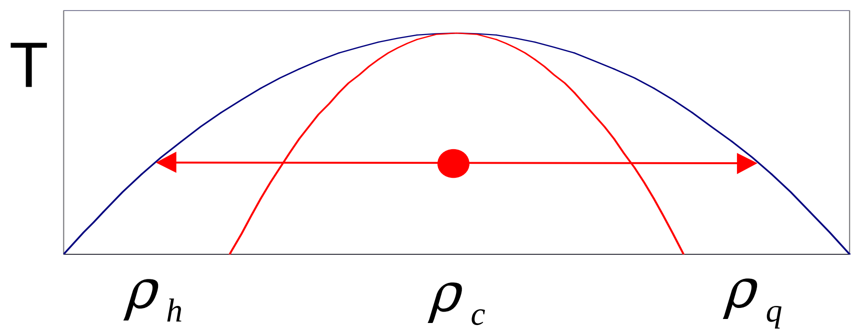
- normally energy independent
- ≈1 if nothing happens

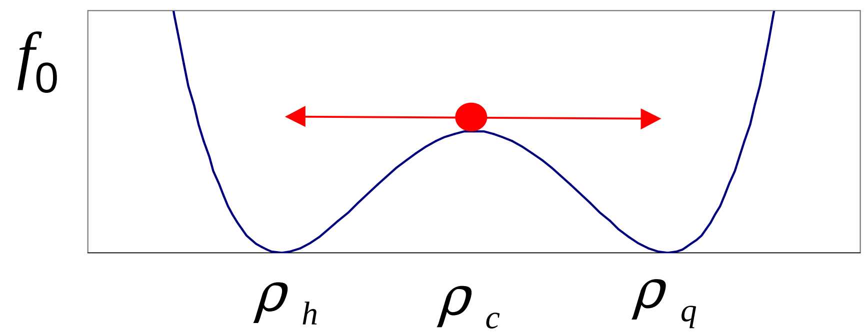



T
unstable
ρ
-0.7
-0.75
-0.8
-0.85
-0.9
20
40
60
τ = 5τ_c
0
-0.25
-0.5
-0.75
τ = 2.5τ_c
0.5
-0.5
τ = 2τ

Phase Separation Dynamics

expansion – quench system deep into coexistence region

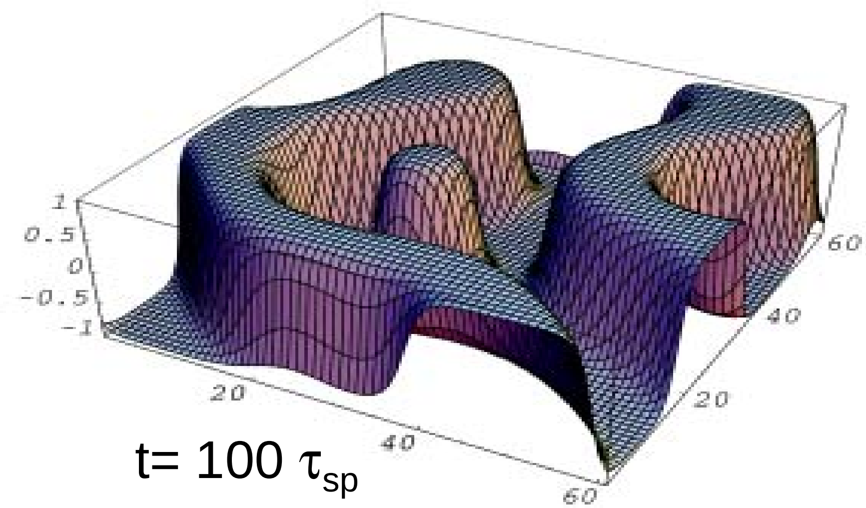




dynamics: $\partial\rho/\partial t = M\nabla^2\left(f_0' + \kappa\nabla^2\rho\right)$

runaway density waves –
fastest growing mode

- time scale $\tau_{sp} \propto \lambda_{sp}^2/D \to 1$ fm
- $\lambda_{sp} \propto$ corr. length $\to 1$ fm
- nonlinear evolution for $t >> \tau_{sp}$



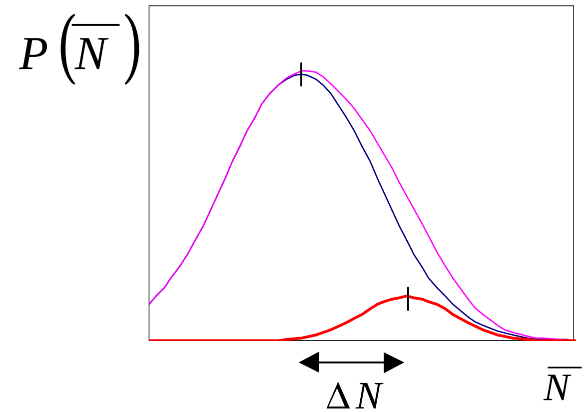
t= 100 τ_{sp}

Event Classes ⇒ Novel Fluctuations

Two event classes with different means

- e.g. **antiprotons**
- fraction f of novel events

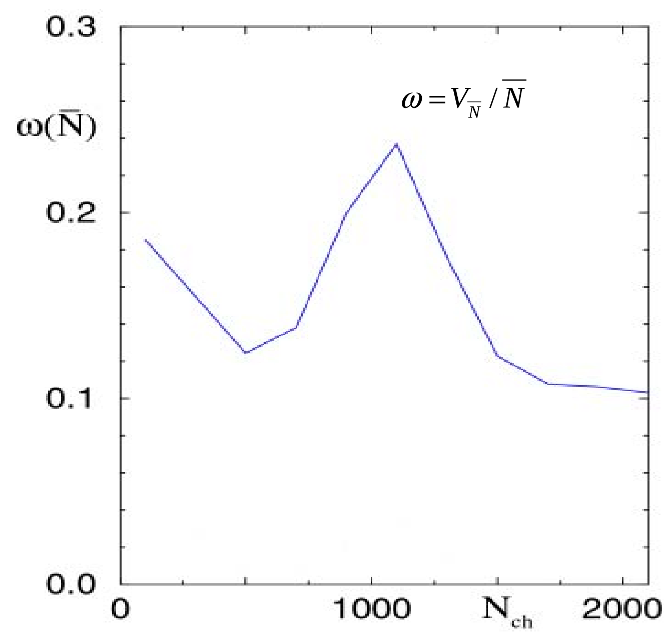
variance : $V \equiv \langle \overline{N}^2 \rangle - \langle \overline{N} \rangle^2 = f(1-f)(\Delta N)^2 + \ldots$



Turn plasma off?
Vary centrality!

Find range where plasma and hadronic classes coexist?

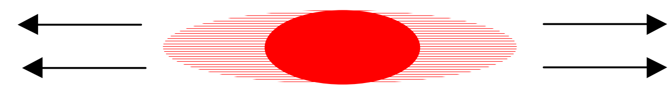
Gavin & Pruneau



To NA49: how does K/π vary with centrality?

Fluctuations Survive Diffusion

expansion stretches



diffusion homogenizes

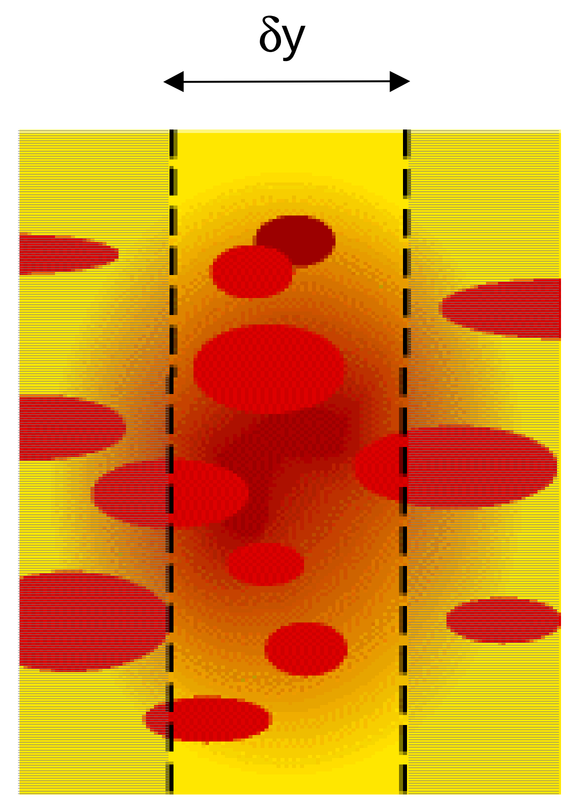
expansion of mesons **fast:** $\delta z \sim \tau$
diffusion **slower:** $\delta z \sim (D\tau)^{1/2}$

expansion **inhibits** rapidity spread -- random walk **limited**

$$\langle \delta y^2 \rangle \sim 2D\left(\frac{1}{\tau_H} - \frac{1}{\tau}\right)$$
$$\rightarrow 2D/\tau_H \sim 1$$

hadronization $\tau_H \sim 10$ fm

diffusion const. $D \sim 2\text{-}10$ fm
Prakash et al.; Heiselberg et al.



Diffusion

diffusion alters baryon current:

$$\partial_\mu j^\mu = 0 \qquad\qquad j^\mu = \rho v^\mu + j^\mu_D$$

- diffusion current $j_D^{\,i} \approx -D\nabla^i\rho$ (local rest frame)
  baryon density ρ, diffusion constant D
- longitudinal flow $v_z \sim z/\tau$
  proper time τ

$$\frac{\partial \rho}{\partial \tau} + \frac{\rho}{\tau} = D\left(\frac{1}{\tau^2}\frac{\partial^2}{\partial \eta^2} + \nabla_\perp^2\right)\rho$$

baryon diffusion in spatial rapidity η

$$N = \delta y \int dr_\perp \rho\tau \qquad \Rightarrow \qquad \tau^2 \frac{\partial N}{\partial \tau} = D \frac{\partial^2 N}{\partial \eta^2}$$

$$\left\langle \Delta\eta^2 \right\rangle = 2D\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right)$$

Phase Separation in Ion Collisions

S. Gavin, nucl-th/9908070; D. Bower and S. Gavin, *in progress*

longtudinal flow + diffusion:

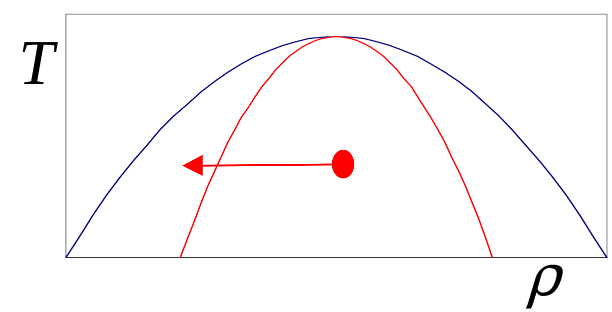
$$\frac{\partial\rho}{\partial\tau}+\frac{\rho}{\tau}=M\nabla^2\left(f_0'+\kappa\,\nabla^2\rho\right)$$

- $\partial_\mu(\rho v^\mu + j_D{}^\mu) = 0$; diffusion current j_D
- Bjorken flow: $v_z \sim z/\tau$, proper time τ

evolution following a quench

- quench at $1 < \tau_c < 10$ fm
- assume $\tau_{sp} \sim$ corr. length ~ 1 fm

drives system out of unstable region



Origin of fluctuations

expansion -- quench system
deep into coexistence region

mechanical instability
$\partial\rho/\partial\mu < 0$

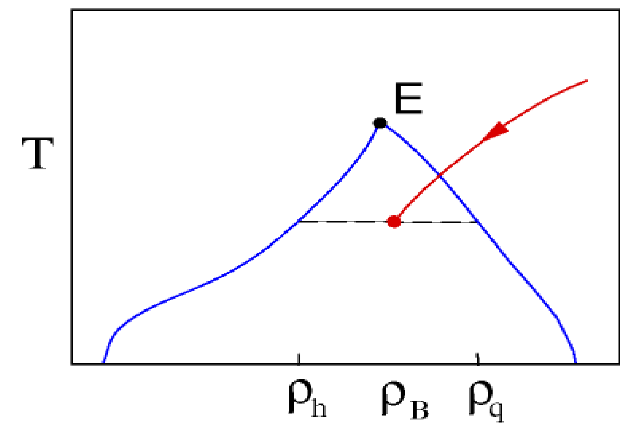
fragmentation

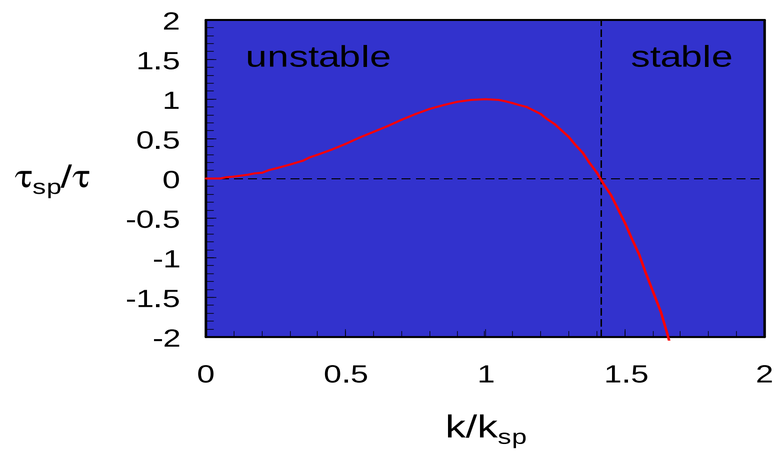
runaway modes
fastest mode
(~ DCC)

$\tau_{sp} \sim (|D|k_{sp})^{-1}$
< 1 fm

$k_{sp}^{-1} \sim m_\sigma^{-1}$

~ bubble size

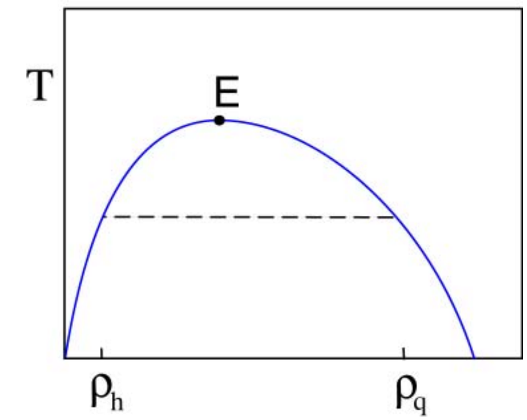


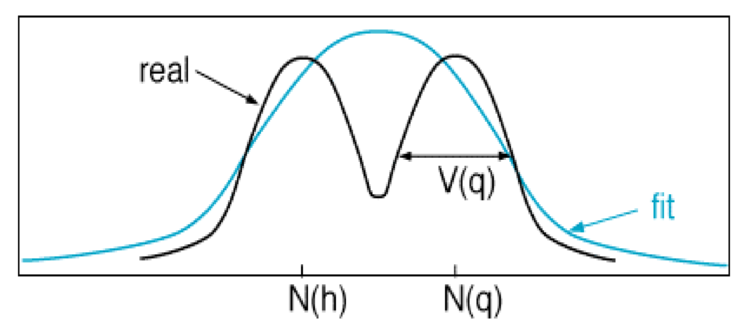


Baryon Number Fluctuations

bimodal distribution of densities within each event "mixed phase"

- plasma fraction *f*
- *N* net baryons in δy





$$V \equiv \langle N^2 \rangle - \langle N \rangle^2$$
$$= f(1-f)\left(N(q) - N(h)\right)^2 + \ldots$$

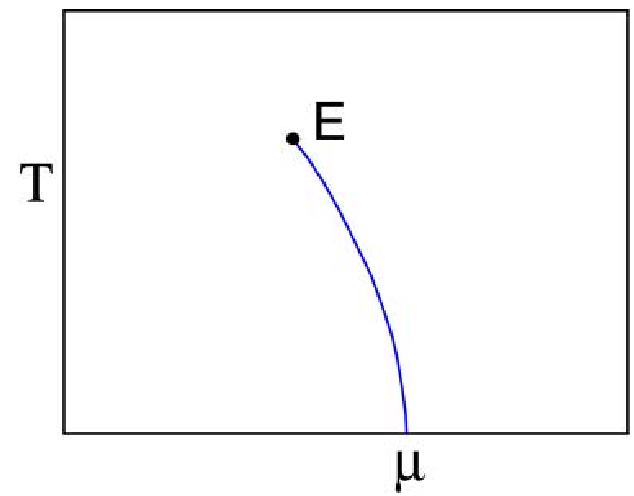
ask: do these fluctuations survive?

1st Order Transition ⇒ Fluctuations

1st order at net baryon density > 0?
Berges & Rajagopal, et al.

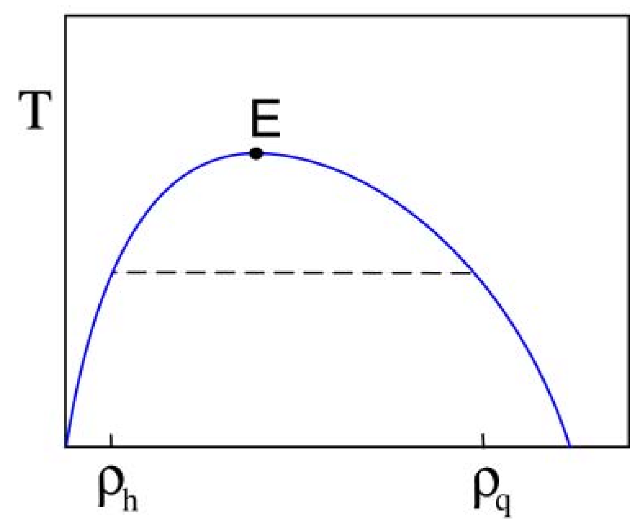
- hadronic bubbles, density ρ_h
  plasma, ρ_q

- **bimodal distribution**
  of densities -- "mixed phase"



fluctuations enhance
variance of baryon number

- plasma fraction f
- N net baryons in Δy



$$V \equiv \langle N^2 \rangle - \langle N \rangle^2 = f(1-f)(\Delta N)^2 + \ldots$$