Fluctuation Probes of Quark Deconfinement

Masayuki Asakawa

Nagoya University

— with Ulrich Heinz (Ohio State)
Berndt Müller (Duke)

Fluctuation Observables

Recently, Fluctuation observables in relativistic heavy ion collisions have attracted a lot of attention in many contexts.

- Fluctuation of pion multiplicity
- Fluctuation of pion slope parameter $T$
- Fluctuation of $N_+/N_-$

However, these hadronic observables are easily modified in hadronic phase.

\[ \Delta^+ \rightarrow p + \pi^0 \] (change of particle number)
\[ p + \pi^- \rightarrow n + \pi^0 \] (change of charged particle number).
Different type of Fluctuations

Fluctuations of locally conserved quantities.

In addition, sensitive to the microscopic structure of the dense matter.

- Survivability?

If the expansion in RHI coll. is too fast for local fluctuations to follow, characteristic features of the plasma phase are expected to survive.

This is a rare occasion where hadronic observables carry information on the initial condition.

- What satisfies the constraint?

  i) Net Baryon Number
  ii) Net Electric Charge
  iii) Net Strangeness

i) and ii) are sensitive to the microscopic structure of matter.

Note again that our proposals are different from previously proposed ones such as fluctuation of \( N_\pi, N_{\text{baryon}} \ldots \) etc.
- More Quantitative Discussion

Generally,

for a conserved quantity $\mathcal{O}$ and the associated chemical potential $\mu$,

$$(\Delta \mathcal{O})^2 = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2 = T \frac{\partial \langle \mathcal{O} \rangle}{\partial \mu}$$

$$\langle \mathcal{O} \rangle = \frac{\text{Tr} \mathcal{O} e^{-(\mathcal{H} - \mu \mathcal{O})/T}}{\text{Tr} e^{-(\mathcal{H} - \mu \mathcal{O})/T}}$$

For $\mathcal{O} = N_b$, $(\Delta \mathcal{O})^2$ is the baryon number susceptibility.

- Net Baryon Number

  - Hadron Gas Phase

    Baryons – heavy, $\mu$ – small enough (SPS, RHIC...),

    $$N_b^\pm(T, \mu) = N_b(T, 0) \exp(\pm \mu/T)$$

    Net Baryon number $N_b = N_b^+ - N_b^-,$

    $$(\Delta N_b)^2_{\text{HG}} = N_b^+ + N_b^- = 2N_b(T, 0) \cosh(\mu/T)$$

    --- rather strong $\mu$ dependence

  - QGP Phase with two massless flavors

    $$\frac{1}{V} (\Delta N_b)^2_{\text{QGP}} = \frac{2}{9} T^3 \left[ 1 + \frac{1}{3} \left( \frac{\mu}{\pi T} \right)^2 \right]$$

    Desirable to normalize with "an extensive and conserved quantity"!
Entropy!

\[
\frac{1}{V} S_{\text{QGP}} = \frac{74 \pi^2}{45} T^3 \left[ 1 + \frac{5}{37} \left( \frac{\mu}{\pi T} \right)^2 \right]
\]

\[
\left( \frac{\Delta N_b}{S} \right)_{\text{QGP}}^2 = \frac{5}{37 \pi^2} \left[ 1 + \frac{22}{111} \left( \frac{\mu}{\pi T} \right)^2 + \cdots \right]
\]

for \( \mu \ll T \), no \( \mu \) dependence.

On the other hand, in HG phase, many resonances exist

\[ \implies S \text{ cannot be written in a simple form.} \]

But, the ratio in HG phase has stronger \( \mu \) dependence.

---

- Net Charge
  - Hadron Gas Phase
    
    Almost all charged hadrons have unit electric charge.

    \[ \implies (\Delta Q)^2_{\text{HG}} = N_{\text{ch}} \text{ (with Boltzmann approximation).} \]

    dominated by pions — weaker \( \mu \) dependence than \( (\Delta N_b)^2_{\text{HG}} \)

  - QGP Phase

    \[
    \left( \frac{\Delta Q}{S} \right)_{\text{QGP}}^2 = \frac{25}{74 \pi^2} \left[ 1 + \frac{22}{111} \left( \frac{\mu}{\pi T} \right)^2 + \cdots \right]
    \]

    Again, weak \( \mu \) dependence.
○ Estimate of $S$ at SPS

We mainly use the NA49 Pb+Pb data.

- $dN_b/dy \sim 92$, anti-baryon/baryon $\sim 0.085$
- Specific entropy at SPS, $S/N_b \sim 36$ (Cleymans, Redlich)

$\implies$ if the fluctuations reflect an equilibrium HG,
\[
\frac{(\Delta N_b)^2}{S} = \frac{(N_b^+ + N_b^-)}{S} \sim 0.033,
\]
while $\frac{(\Delta N_b)^2}{S} \sim 0.014$ in QGP.

○ Charge Fluctuation

Experimentally, $\frac{dN_{ch}}{dy} \sim 400$.

However, this needs a correction.

About 60% of final state pions are from resonance decays. (Sollfrank, Koch, Heinz)

$\implies (\Delta Q)^2 / S \sim 0.06$ (HG)
\[
\sim 0.036 \text{ (QGP)}
\]

$(\Delta N_b)^2 / S$ and $(\Delta Q)^2 / S$ at RHIC/LHC were estimated with hadrochemical models (Braun-Munzinger, Stachel, Zimányi...)
with the chemical freeze-out temperature 170 MeV.
Is this difference (~ factor 2) observable?

- Even if a QGP is temporarily created, all hadrons are emitted after phase transition.
  
  # of charged particles...etc. — not conserved
  e.g., \( gg \rightarrow \pi^+ \pi^- \)

- The fluctuations of conserved quantum numbers can be changed only by particle transport.

- For simplicity, let us assume the fireball expands longitudinally with a boost-invariant (Bjorken) flow profile.

\[
\begin{align*}
\tau = \text{const} \\
z = -t \\
z = t
\end{align*}
\]

In the following, we assume \( T_i = 170 \text{ MeV}, \ T_f = 120 \text{ MeV} \).

(phase transition) (kinetic freezeout)

At RHIC, e.g., \( \tau_i \sim 5 \text{ fm}, \ \tau_f \sim 14 \text{ fm} \).
Qualitative Discussion

In actual measurement, we have to observe a subvolume, not the total system.

Suppose we take a rapidity interval $\Delta \eta \sim 1$.

![Diagram showing $\Delta \eta = 1$ and spatial distances 5 fm to 14 fm]

Baryons have average thermal longitudinal velocity

$$\bar{v}_z = \sqrt{\frac{2T}{\pi M}} \approx 0.33 \quad (M = 1 \text{ GeV}, \ T = 170 \text{ MeV})$$

Thus, between $\tau_i$ and $\tau_f$, a baryon which was initially at the center can travel on average only 3 fm in $z$ direction.

Rescattering effect is not taken into account.

$\Rightarrow$ Actual diffusion is much slower.
More Quantitative Discussion

i) Smearing of Fluctuation

\[ \Delta \eta \]

Difference in the baryon densities inside and outside the subvolume

\[ \implies \text{Difference in the mean flux of baryons into and out of the volume.} \]

ii) Fluctuation in the baryon fluxes exchanged with the neighboring subvolume.

\[ (\Delta N_b)_{HG}^2 = N_b^+ + N_b^- \]

\[ \Delta N_b = \Delta N_b^{(i)} \exp \left( -\frac{3\bar{v}_i}{\Delta \eta} [1 - (T_f/T_i)^{1/2}] \right) \]

\[ \sim \Delta N_b^{(i)} \exp \left( -\frac{\bar{v}_i}{2\Delta \eta} \right) \]

\[ \implies \Delta \eta \geq \frac{\bar{v}_i}{2} \sim 0.33 \]

\[ N_b^{(en)} = N_b^{(lv)} \sim \frac{N_b^{(i)} \bar{v}_i}{2\Delta \eta} \]

\[ \frac{(\Delta N_b^{(ex)})^2}{(\Delta N_b^{(i)})^2} \sim \frac{\bar{v}_i}{\Delta \eta} \]

\[ \implies \Delta \eta \geq \bar{v}_i \sim 0.65 \]
Comments

- Again, short mean free path effect will reduce this estimate.
- During hadronization cooling is actually slowed down.
- However, transverse expansion will reduce the time scale substantially.

Expected to partially cancel each other

Thus, $\Delta \eta \geq 1$ (equivalently, $\Delta y \geq 1$) will be sufficient to observe the difference.

Furthermore,

- If QGP phase is gluon dominated as in the hot glue scenario, (Shuryak, Geiger...)

$$(\Delta N_b)^2/S, (\Delta Q)^2/S \quad \text{Further reduced in QGP phase}$$
Further Comments I

- Relation between $Q$ fluctuation and $N_+ / N_-$ fluctuation?

For $R = \frac{N_+}{N_-}$,

$$N_{ch} (\Delta R)^2 \sim \frac{4(\Delta Q)^2}{N_{ch}} \propto \frac{(\Delta Q)^2}{S}$$

to the leading order and

if $N_{ch}$ is not changed in the hadronic phase (⇒ Koch's talk).

- Rescattering Effect?

Rescattering (mainly $\pi\pi \rightarrow \rho \rightarrow \pi\pi$, $\pi N \rightarrow \Delta \rightarrow \pi N$)

accelerates the relaxation of fluctuations? (Shuryak and Stephanov)
Conclusions

1. Local fluctuations of the net baryon number and electric charge differ in HG and QGP.

2. The difference is partially frozen at early stage in RHI collisions.

3. Thus, these fluctuations may be useful probes of the formation of a deconfined phase, i.e., QGP.

4. Measure event by event fluctuations of the two quantities at a fixed $dE_{T}/dy$ or energy deposited in ZDC.

5. In the QGP scenario, $(\Delta Q)^2/(\Delta N_b)^2 = 5/2$, while it is beam energy dependent in the HG scenario.