$E_\perp$ as a Barometer for a Saturated Plasma

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- Gluon Minijet Production and the Saturation Model / Assumption
- Transport Theory with Saturation Initial Conditions
- $dE_\perp/dy$ as a Barometer, the Effective Longit. Pressure in the Saturated Plasma
- Results and Discussion

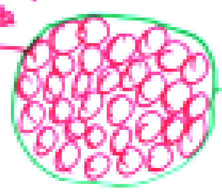
Gluon Minijet Production in A+A and Saturation...

("Naive" Geometrical Version Deliberately Stolen From EKRT: NPB 570, 2000)

$$\frac{dN^g}{dy}(p_0) = K\, T_{AA}(b) \int_{p_0}^{\infty} d^2p_\perp \int_0^1 dx_a \int_0^1 dx_b\, G(x_a,Q^2)\, G(x_b,Q^2)\, \frac{\hat{s}}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}\, \delta(\hat{s}+\hat{t}+\hat{u})$$

Now define p_{sat} by $\quad \frac{dN^g}{dy}(p_{sat}) = \frac{p_{sat}^2}{\pi}\, \frac{\pi R^2}{\beta}$

gluon "area" π/p_{sat}^2



nuclear $\perp$ area

(EKRT: $\beta = 1$; A. Mueller: $\beta = \frac{4\pi\alpha(p_{sat}) N_c}{c\,(N_c^2-1)}$)

for $\beta = 1 \to p_{sat} = 0.208\, A^{0.128} \sqrt{s}^{\,0.191}$ $\quad$ ~ 1.2 GeV at RHIC, ~ 2.2 GeV at LHC $\}$ $A \sim 200$

$\boxed{p_{sat} \text{ runs with } A \text{ and } \sqrt{s}}$

Huge Minijet Densities at time $\tau_0 = 1/p_{sat}$!

$$s_0 = \frac{1}{\pi} p_{sat}^3, \quad \varepsilon_0 = \frac{C_1}{\pi} p_{sat}^4, \quad \frac{dN^g}{dy} \sim \begin{matrix} 1000 & \text{RHIC} \\ 3000 & \text{LHC} \end{matrix} \;!$$

Is there also a huge pressure $p \sim \varepsilon/3$ as in lattice QCD ??

How fast does the pressure approach the LQCD value if starting from $p_\parallel(\tau_0) = 0$??

→ For field-theoretical motivation of saturation in p & A @ xccl see works of Blaizot/Mueller, Kovchegov/Mueller, Frankfurt/Strikman & in particular the classical YM theory of McLerran/Venugopalan, Venug./Krasnitz, Iancu/Leonidov/McLerran, ...

Measure The Pressure (longitudinal) via $\frac{dE_\perp}{dy}$:

Energy-Momentum Conservation: $\boxed{\partial_\mu T^{\mu\nu} = 0}$

Perfect Fluid: $T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}$

$$\curvearrowright \quad \dot{\varepsilon} = -(\varepsilon + p)\, \partial \cdot u$$

1d Hubble flow (Bjorken) $\partial \cdot u = \frac{1}{\tau}$, $\tau = \sqrt{t^2 - z^2}$

$$\curvearrowright \quad \boxed{\frac{\tau \varepsilon}{\tau_0 \varepsilon_0} = \left(\frac{\tau_0}{\tau}\right)^{\delta}}$$

* for local thermodynamical equilibrium $\delta = c_s^2$ (= isentropic speed of sound)
  $\left(\Gamma_{scatt} = \frac{1}{\rho\, \sigma_{tr.}} \gg \partial \cdot u\right)$

* for free streaming $\delta = 0$ $\quad \left(\Gamma_{scatt.} \ll \partial \cdot u\right)$

Experimental Observable:

$$E = \int d\sigma_\mu T^{\mu 0}$$

$$d\sigma_\mu u^\mu = \tau\, d^2 x_\perp\, dy$$

$$\boxed{\frac{dE_\perp}{dy} = \tau \varepsilon\, \pi R_A^2}$$

Transport

Perturbative (!) Rescattering
a la Geiger/Müller NPB 369, 1992.
(very simplified version...)

$$p\cdot\partial f(x,p) = \Gamma_{rel}(p\cdot u)\left[f_{eq}(p\cdot u) - f(p,x)\right]$$

Relaxation Rate $\Gamma_{rel} = \frac{1}{E}\frac{dE}{dz} = \rho\int\left(d\sigma_{el}\frac{\Delta E_{el}}{E} + d\sigma_{in}\frac{\Delta E_{in}}{E}\right)$

$$\approx \rho\int_{\mu^2}^{Q^2} dq^2\,\frac{d\sigma_{el}}{dq^2}\left\{\frac{q^2}{2E^2} + \frac{\alpha N_c}{\pi}\int_{\mu^2}^{q^2}\frac{dk_\perp^2}{k_\perp^2}\int_0^1\frac{dx}{x}\,x\right\}$$

[μ^2: Debye screening scale
($= N_c g^2 T^2/3$ in therm. equilibrium)]

$$= \rho\left(\frac{16\pi\alpha^2 N_c^2}{N_c^2-1}\right)\left(\frac{1}{s}\log\frac{Q^2}{\mu^2} + \frac{\alpha N_c}{\pi\mu^2}\log\frac{\overline{q^2}}{\mu^2}\right)$$

[$\overline{q^2} \approx \mu^2\log\frac{Q^2}{\mu^2}$
$s/\mu^2 \approx \frac{9}{2\pi\alpha}$]

$$\approx 9\pi\alpha^2\frac{\rho^3}{\varepsilon^2}\left(\log\frac{1}{\alpha} + \frac{27}{2\pi^2}\right) \equiv K_{in}\,9\pi\alpha^2\frac{\rho^3}{\varepsilon^2}\log\frac{1}{\alpha}$$

for example at $\tau = \tau_0 = 1/p_{sat}$: $\frac{\Gamma_{rel}}{\partial\cdot u} \approx K_{in}\frac{9\alpha^2}{\beta}\log\frac{1}{\alpha}$

$\to 0$ as $p_{sat}\to\infty$! $(\sqrt{s}, A\to\infty)$

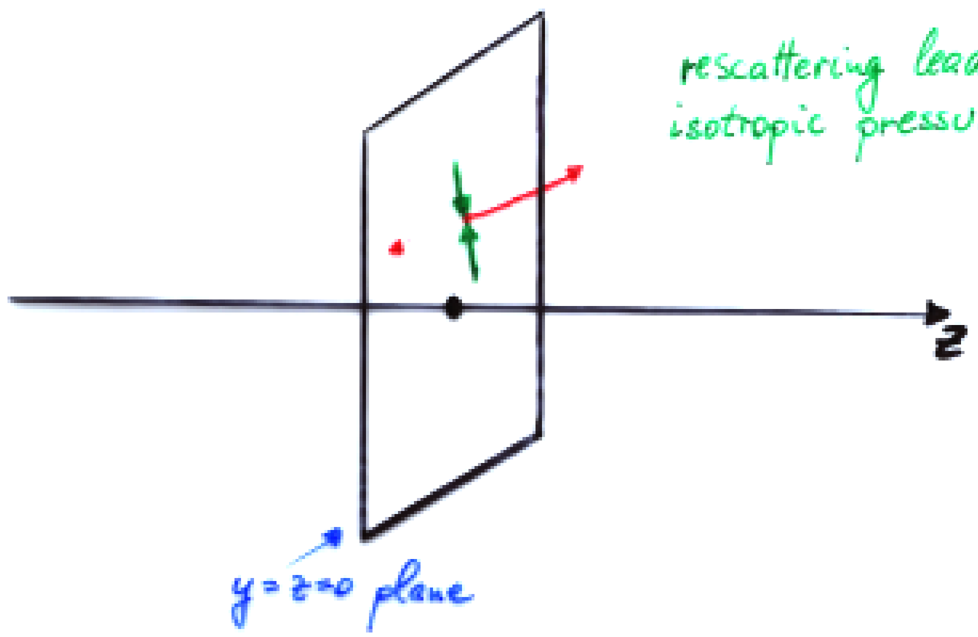
To follow the time evolution integrate Boltzmann Eq. numerically:

$$e^{\phi}\,\frac{\tau\varepsilon}{\tau_0\varepsilon_0} = 1 + \underbrace{\int_0^{\phi} d\phi'\, e^{\phi'}\,\frac{\tau'(\phi')\,\varepsilon(\phi')}{\tau_0\,\varepsilon_0}\, h\!\left(\frac{\tau'(\phi')}{\tau(\phi)}\right)}_{\text{Collision Integral}}$$

see e.g. Baym, PLB 138, 1984

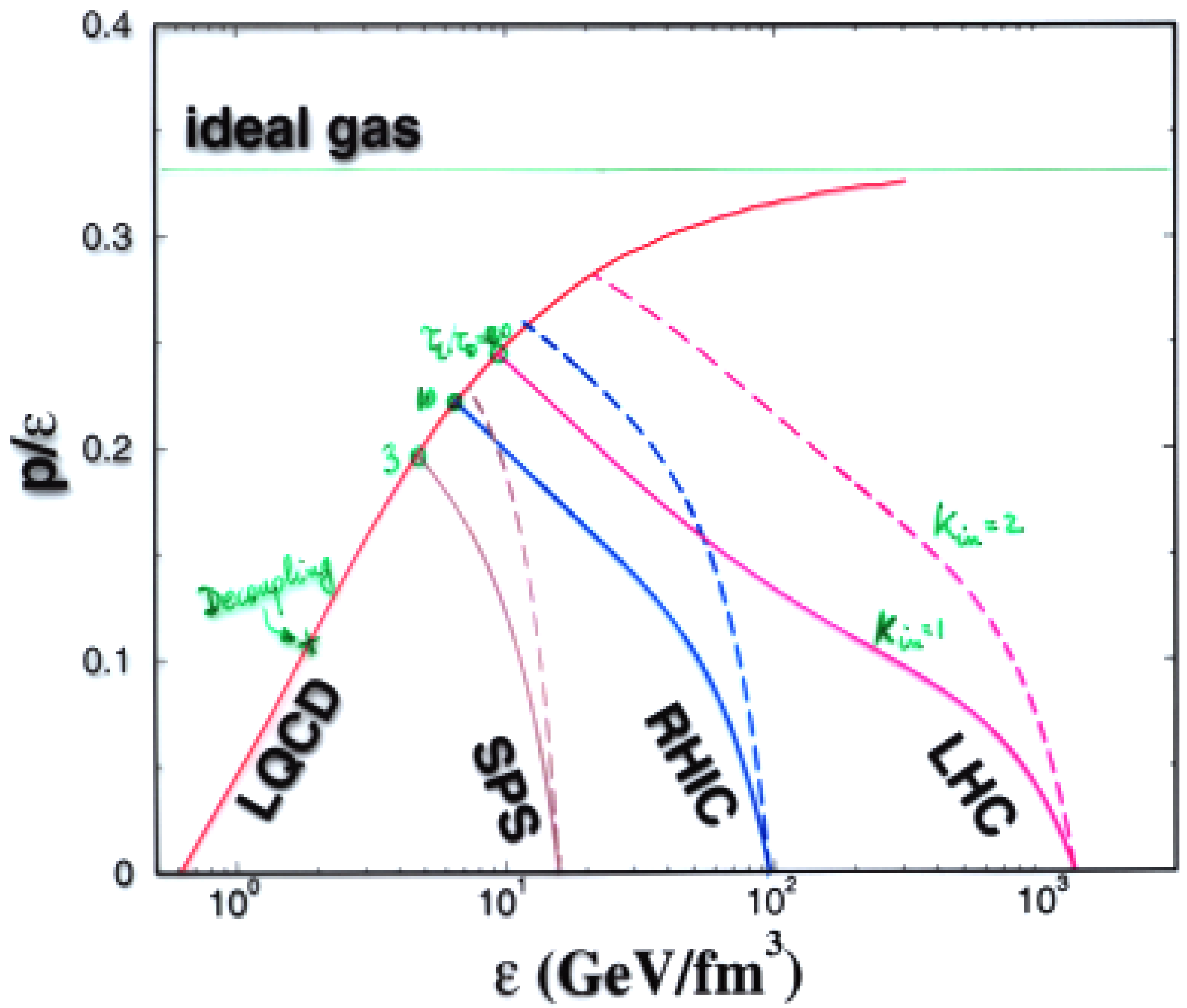
$$\phi(\tau) = \int_{\tau_0}^{\tau} d\tau'\, \Gamma_{\text{rel}}(\tau')$$ # of collisions in time interval $[\tau_0, \tau]$

$$h(x) = \frac{1}{2\sqrt{1-x^2}}\left(x\sqrt{1-x^2} + \arcsin\sqrt{1-x^2}\right)$$



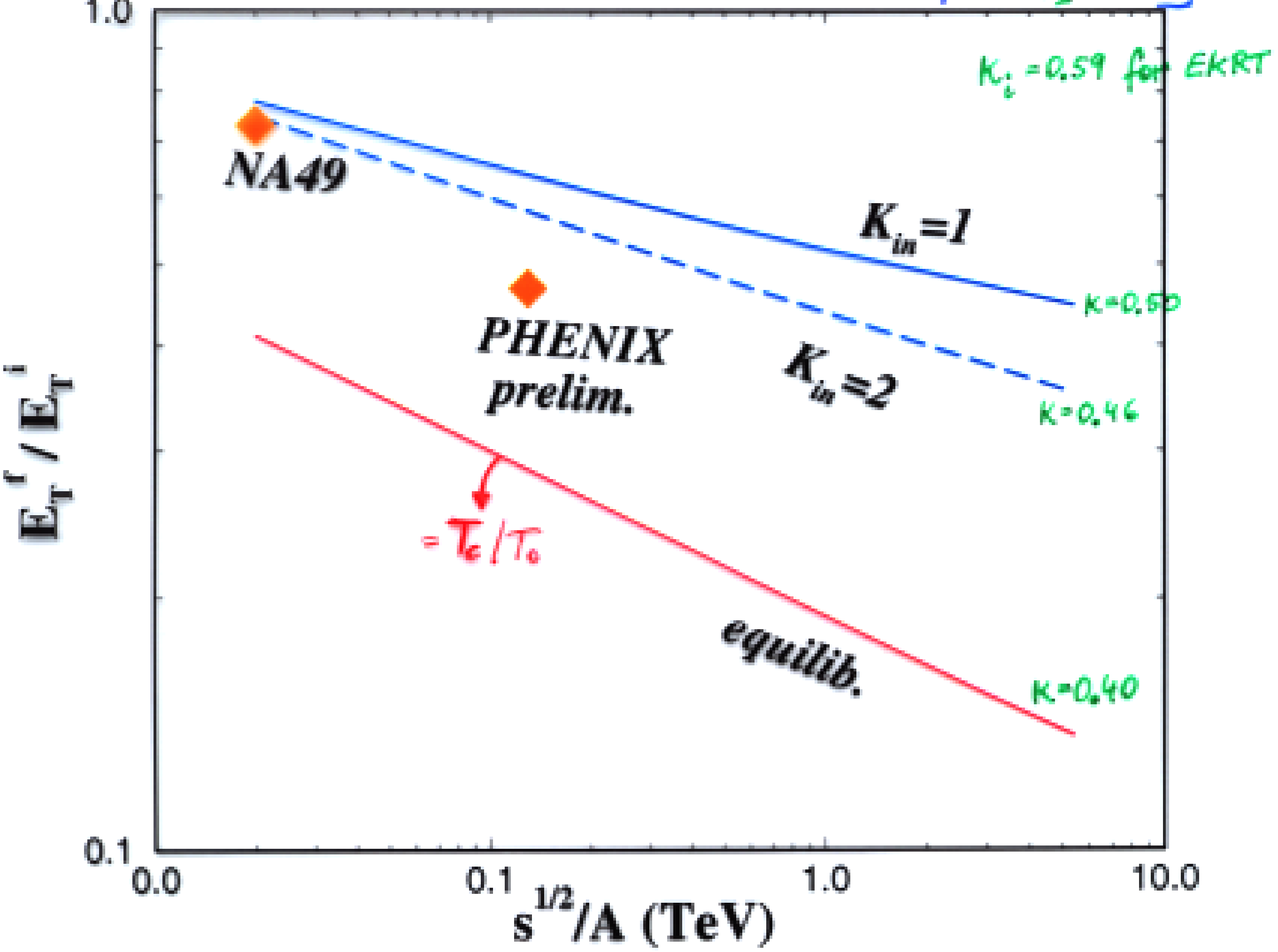
rescattering leads to more isotropic pressure

- Initially longitudinal pressure $\simeq 0$ in case of Bjorken-type space-momentum correlation
- LQCD pressure is reached at time τ_L; τ_L/τ_0 increases with energy because p_{sat} increases!
- for $\tau > \tau_L$ assume evolution along LQCD curve, decoupling at $\varepsilon_f \simeq 2\ \mathrm{GeV/fm^3}$ at time $\tau_f = \tau_L \left(\frac{1 + a + b \log \varepsilon_L}{1 + a + b \log \varepsilon_f} \right)^{1/b}$



- $E_\perp(\sqrt{s})$:
  - → $E_\perp^i = \tau_0 \epsilon_0 \pi R^2 = \frac{C_1}{\pi} p_{sat}^3 \pi R^2 \sim (\sqrt{s})^{0.59}$ for EKRT
- $E_\perp^f$ depends on the amount of final-state interactions
  - → maximal $E_\perp$-loss from ideal Euler hydro, $E_\perp^f = E_\perp^i \frac{T_f}{T_i} \sim (\sqrt{s})^{0.40}$
  - → purely elastic $gg \to gg$ (i.e. $K_{in}=1$) is inbetween, gives $E_\perp^f \sim (\sqrt{s})^{0.50}$
- $K_{DATA} < 0.40$ from 17.4 AGeV to 130 AGeV ! ↯ present theory is quantitatively inaccurate in this energy domain
- Data at $\sqrt{s} = 56, 200$ needed...

$$\boxed{K = \frac{\sqrt{s}}{E_\perp^f} \frac{dE_\perp^f}{d\sqrt{s}}}$$



SUMMARY

- Saturation or not affects Minijet evolution after τ_0!
  (QGP formation, density, temperature, pressure ...)
  - p_0 fixed (HIJING, Parton Cascade, ...):
    $\tau_0 = 1/p_0$ fixed, Minijet Multiplicity Increases with $\sqrt{s}$, A
    → final state interactions asymptotically approach ideal, non-viscous hydro
  - $p_0(\sqrt{s}, A)$ increasing (Saturation Model): $\alpha(p_0)$ decreases,
    - perturbative rescattering becomes weaker!
    - approach towards pressure of QCD in equilibrium (LQCD) slows down
    - final state may reveal more closely scaling properties of saturation with A, $\sqrt{s}$
      e.g. $dE_\perp/dy$
- More detailed/accurate transport studies with saturation initial conditions must be done
  (determine pressure, T, e-density relative to LQCD)

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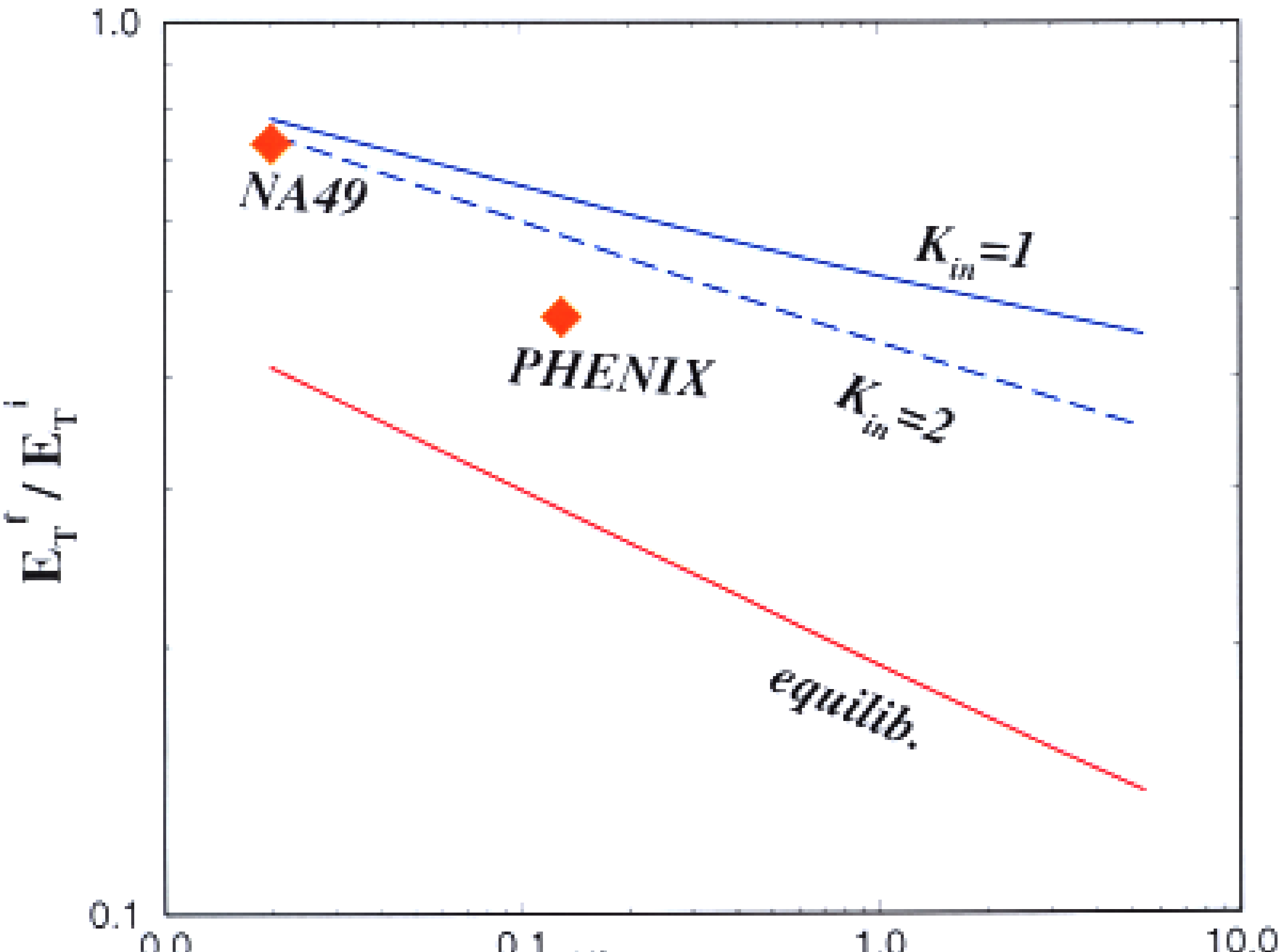

1.0
0.1
0.0
0.1
1.0
10.0
NA49
PHENIX
K_in=1
K_in=2
equilib.
E_T^f / E_T^i
s^1/2/A (TeV)