$E_1$ as a Barometer for a Saturated Plasma

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hep-ph/0006257

- Gluon Minijet Production and the Saturation Model / Assumption
- Transport Theory with Saturation Initial Conditions
- $dE/dy$ as a Barometer, the Effective Longitudinal Pressure in the Saturated Plasma
- Results and Discussion
Gluon Minijet Production in A+A and Saturation... ("Naive" Geometrical Version Deliberately Stolen From EKRT: NPB 570, 2000)

\[
\frac{dN^g}{dy}(p_0) = K T_{AA}(b) \int_0^\infty d^2 p_z \int_0^1 dx_a \int_0^1 dx_b \frac{G(x_a, Q^2) G(x_b, Q^2)}{x_a x_b} \frac{e}{\pi} \frac{d\sigma}{dE} \sigma(2\pi R^2/nuclear\text{\ area})
\]

Now define \( P_{\text{sat}} \) by

\[
\frac{dN^g}{dy}(P_{\text{sat}}) = \frac{P_{\text{sat}}^2}{\pi} \frac{R^2}{r^2}
\]

(EKRT: \( \beta = 1 \); A. Mueller: \( \beta = \frac{4\pi \alpha(\text{\ energy\ at\ RHIC}) N_c}{c (N_c^2 - 1)} \))

for \( \beta = 1 \rightarrow P_{\text{sat}} = 0.208 A^{0.128} \sqrt{S^{0.191}} \sim 1.2 \text{ GeV at RHIC} \sim 2.2 \text{ GeV at LHC} \)

\( P_{\text{sat}} \) runs with \( A \) and \( \sqrt{S} \)

Huge Minijet Densities at time \( t_0 = 1/P_{\text{sat}} \)

\( S_0 = \frac{1}{\pi} P_{\text{sat}}^3 \), \( E_0 = \frac{c}{\pi} P_{\text{sat}}^4 \), \( \frac{dN^g}{dy} \sim 1000 \text{ RHIC} \sim 3000 \text{ LHC} \)

Is there also a huge pressure \( P \sim \epsilon/3 \) as in lattice QCD ??

How fast does the pressure approach the LQCD value if starting from \( P_{\text{H}}(t_0) = 0 \) ??

For field-theoretical motivation of saturation in p+A see works of Blaizot/Mueller, Kovchegov/Mueller, Frankfurt/Strickman & in particular the classical YM theory of Meliclan/Venugopalan, Veneziano/Kraski, Iancu/Leonidov/Meliclan, ...
Measure the Pressure (longitudinal) via $\frac{dE_L}{dy}$:

Energy - Momentum Conservation: $\partial_j T^{00} = 0$

Perfect Fluid: $T^{\mu\nu} = (\varepsilon + p) u^{\mu} u^{\nu} - pg^{\mu\nu}$

$\Delta \varepsilon = - (\varepsilon + p) \Delta u$

1d Hubble flow (Bjorken) $\Delta u = \frac{1}{c}, \tau = \sqrt{t^2 - z^2}$

$\frac{\Delta \varepsilon \tau}{\varepsilon_0 c_0} = \left( \frac{\tau_0}{c} \right)^2$

* for local thermodynamical equilibrium $\delta = \frac{\Delta \varepsilon}{\varepsilon} \approx \text{isentropic speed of sound}$

$\left( \frac{T_{\text{scat}}}{\varepsilon} \approx \frac{1}{c} \gg \Delta u \right)$

* for free streaming $\delta = 0 \quad (T_{\text{scat}} \ll \Delta u)$

Experimental Observable:

$E = \int d\sigma \tau T^{00}$

$\sigma_{\mu\nu} = \tau d^3x_\perp dy$

$\frac{dE_L}{dy} = \tau \varepsilon \pi R_a^2$
Transport

Perturbative (!) Rescattering
a la Geiger/Müller NPB 369, 1992.
(very simplified version...)

\[ p \cdot \partial f(x,p) = T_{\text{rel}}(p,u) \left[ f_{\text{eq}}(p,u) - f(p,x) \right] \]

Relaxation Rate

\[ T_{\text{rel}} = \frac{1}{\epsilon} \frac{dE}{dt} = \epsilon \left( \sigma_{\text{ee}} \frac{\Delta E_{\text{ee}}}{E} + \sigma_{\text{in}} \frac{\Delta E_{\text{in}}}{E} \right) \]

\[ \approx \epsilon \int \frac{Q^2}{\mu^2} \frac{dE_{\text{ee}}}{dQ^2} \left\{ \frac{Q^2}{2E^2} + \frac{\alpha N_c}{\pi} \int \frac{K^2}{K^2 + \mu^2} \int \frac{dx}{x} \right\} \]

\[ \mu^2: \text{Debye screening scale} \]
\[ (= N_c g^2 T^{2/3} \text{ in therm. equilibrium}) \]

\[ \approx \epsilon \left( \frac{16\pi \alpha^2 N_c^2}{N_c^2 - 1} \right) \left( \frac{1}{5} \log \frac{Q^2}{\mu^2} + \frac{\alpha N_c}{\pi \mu^2} \log \frac{Q^2}{\mu^2} \right) \]

\[ \log \frac{Q^2}{\mu^2} = \frac{Q^2}{\mu^2} - \frac{1}{2} \log \frac{Q^2}{\mu^2} \]

\[ s/\mu^2 = \frac{9}{2\pi \alpha} \]

\[ \approx 9\pi \alpha^2 \frac{S^3}{E^2} \left( \log \frac{1}{\alpha} + \frac{27}{2\pi^2} \right) \equiv K_{\text{in}} 9\pi \alpha^2 \frac{S^3}{E^2} \log \frac{1}{\alpha} \]

for example at \( \tau = t_0 = \frac{1}{\text{P} \text{e} \text{m}} \):

\[ \frac{T_{\text{rel}}}{\delta E/u} = K_{\text{in}} \frac{9\alpha^2}{\beta} \log \frac{1}{\alpha} \]

\( \rightarrow 0 \) as \( \text{P} \text{e} \text{m} \rightarrow 0 \) \( (S^3, \Lambda \rightarrow \infty) \)
To follow the time evolution integrate Boltzmann Eq. numerically:

\[ e^{\frac{\tau_e}{\tau_0}} \frac{\tau_e}{\tau_0} = 1 + \int_0^\phi d\phi' e^{\phi'} \frac{\tau'(\phi') \varepsilon(\phi')}{\tau_0 \varepsilon_0} h\left(\frac{\tau'(\phi')}{\tau(\phi)}\right) \]

**Collision Integral**

\[ \phi(\tau) = \int_{\tau_0}^{\tau} d\tau' n_{ee} (\tau') \quad \# \text{ of collisions in time interval } [\tau_0, \tau] \]

\[ h(x) = \frac{1}{2 \sqrt{1-x^2}} \left( x \sqrt{1-x^2} + \arcsin \sqrt{1-x^2} \right) \]

Rescattering leads to more isotropic pressure.
- Initially longitudinal pressure $\propto$ in case of Bjorken-type space-momentum correlation

- LQCD pressure is reached at time $\tau_L$, $\tau_L/\tau_0$ increases with energy because $p_{\text{sat}}$ increases!

- for $\tau > \tau_L$ assume evolution along LQCD curve, decoupling at $\varepsilon_f \approx 2 \text{ GeV/fm}^3$ at time $\tau_f = \tau_L \left( \frac{1 + a + b \log \varepsilon_f}{1 + a + b \log \varepsilon_f} \right)^{1/b}$
$E_\perp^i$: 

- $E_\perp^i \sim T_0 \varepsilon_0 \pi R^2 = \frac{C_i}{\pi} P_{\text{sat}}^3 \pi R^2 \sim (\beta^2)^{0.59}$ for EKRT

- $E_\perp^f$ depends on the amount of final-state interactions
  - maximal $E_\perp$-loss from ideal Euler hydro, $E_\perp^f = E_\perp^i \frac{T_0}{T_f} \sim (\beta^2)^{0.40}$
  - purely elastic $gg \rightarrow gg$ (i.e. $K_{in} = 1$) is inbetween, gives $E_\perp^f \sim (\beta^2)^{0.50}$

- $K_{\text{DATA}} < 0.40$ from 17.4 AGeV to 130 AGeV! Present theory is quantitatively inaccurate in this energy domain
- Data at $\sqrt{s} = 56, 200$ needed...

\[ K = \frac{\sqrt{s}}{E_i^f} \frac{dE_\perp^f}{d\sqrt{s}} \]

\[ K_i = 0.59 \text{ for EKRT} \]
SUMMARY

- Saturation or not affects Minijet evolution after $t_0$!
  (QGP formation, density, temperature, pressure...)

- $P_0$ fixed (Hijing, Parton Cascade, ...):
  $t_0 = \frac{1}{P_0}$ fixed, Minijet Multiplicity Increases with $N, A$
  $\rightarrow$ final state interactions asymptotically approach ideal, non-viscous hydro

- $P_0 (N, A)$ increasing (Saturation Model): $\alpha (P_0)$ decreases,
  - perturbative rescattering becomes weaker!
  - approach towards pressure of QCD in equilibrium (LQCD) slows down
  - final state may reveal more closely scaling properties of saturation with $A, T$
    e.g. $dE/dy$

- More detailed/accurate transport studies with saturation initial conditions must be done
  (determine pressure, T, e-density relative to LQCD)

Thanks to Organizers of QM '01, to my collaborator Millos,
and to Keijo Kajantie, Kari Eskola, Kimmo Toomainen, Jeff Bjorken,
Larry McLerran, Raju Venugopalan, Dan Thanh Son
for discussions & comments.