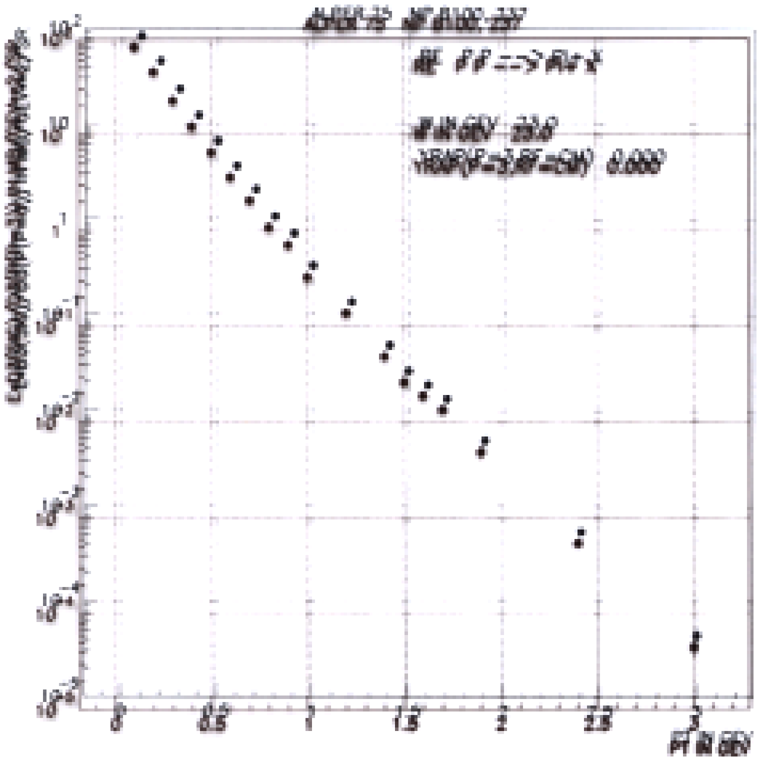
Remarks on the Extraction of Freeze-Out Parameters

Dirk H. Rischke

Brookhaven National Laboratory
University Frankfurt

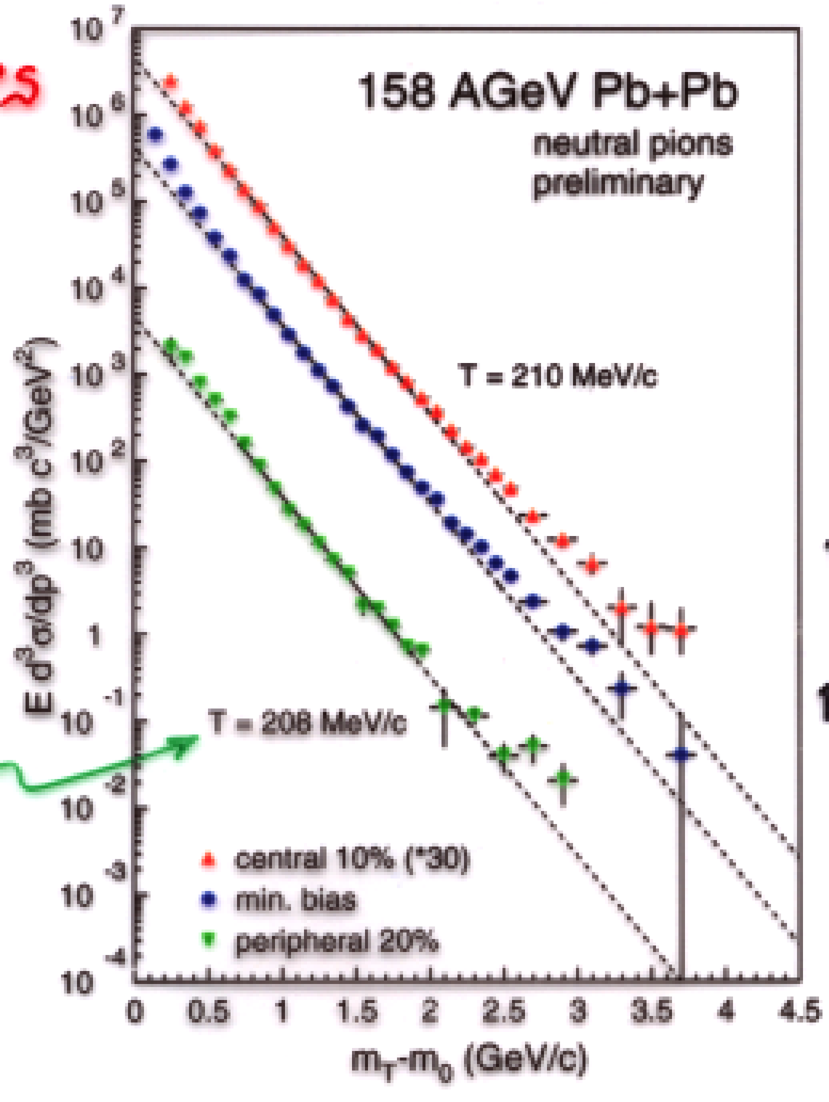
Single inclusive particle spectra

CERN-ISR



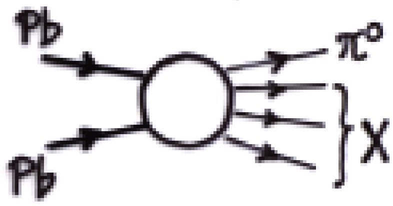
B. Alper et al.
NPB 100, 237 (75)

CERN-SPS



T. Peitzmann
(WA98 coll.)
talk at
QM'97,

http://qgp.uni-muenster.de/WA98/qm97



inverse
slope

$\sim e^{-m_T/T}$

$$\boxed{\text{Thermal equilibrium} \Rightarrow \text{Exponential } m_T\text{-spectra}}$$

<u>Proof</u>: particle no. in thermal system:

$$N = gV \lambda \int \frac{d^3\vec{p}}{(2\pi)^3}\, e^{-E/T}$$

fugacity $\to \lambda$, temperature $\to T$

$E = \sqrt{p^2+m^2} = m_T \operatorname{ch} y$

$$\Rightarrow \quad E\frac{dN}{d^3\vec{p}} \sim \lambda\, e^{-E/T} \quad , \text{ q.e.d.}$$

<u>Note</u>: Inverse slope is determined by T,
Normalization —— " —— λ.

Unfortunately,

$$\boxed{\text{Exponential } m_T\text{-spectra} \not\Rightarrow \text{Thermal equilibrium}}$$

<u>Example</u>: Phase space of $2 \to N$ process, and not the dynamics, gives rise to exponential m_T-spectra!

Proof: borrow ideas from

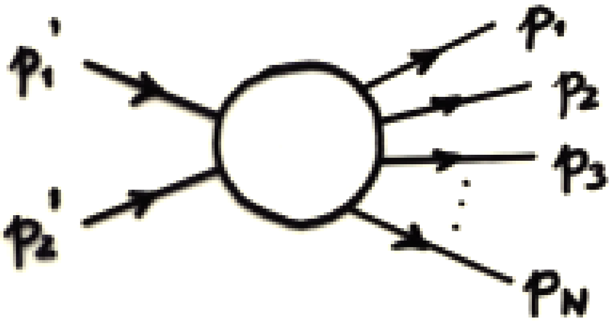
R. Hagedorn, "Relativistic Kinematics", W.A. Benjamin, 1963

E. Byckling, K. Kajantie, "Particle Kinematics", Wiley, 1973

R. Hagedorn in: "Hot Hadronic Matter: Theory and Experiment", eds. J. Letessier et al., Plenum Press, 1995

J. Hormuzdiar, S.D.H. Hsu, G. Mahlon, nucl-th/0001044 ...

$P^\mu = p_1'^\mu + p_2'^\mu$



... and then talk to Keijo !!

$$E_1 \frac{d\sigma}{d^3\vec{p}_1} \sim \underbrace{\int \prod_{i=2}^{N} \frac{d^3\vec{p}_i}{E_i}\, \delta^{(4)}\Big(P - p_1 - \sum_{i=2}^{N} p_i\Big)}_{\equiv\, \Phi(P-p_1)} \underbrace{|\mathcal{M}(P, p_1, \dots, p_N)|^2}_{\text{matrix element}}$$

$$\equiv \Phi(P - p_1)\, \big\langle |\mathcal{M}(P, p_1, \dots, p_N)|^2 \big\rangle_{p_2, \dots, p_N}$$

$$\downarrow$$

$$E_1 \frac{d\sigma}{d^3\vec{p}_1} \equiv \underbrace{\Phi(P - p_1)}_{\substack{\text{contains} \\ \text{phase space} \\ \text{information only}}} \quad \underbrace{\Delta(P, p_1)}_{\substack{\text{contains dynamical} \\ \text{information about} \\ 2 \to N \text{ process}}}$$

Compute $\Phi(P-p_1)$:

Φ is Lorentz-invariant, $\dim \Phi = [\mathrm{MeV}]^{2(N-3)}$.

For $m_1 = \ldots = m_N = 0$,

CM frame, $P^\mu = (E, \vec{0})$

$$\boxed{\Phi(P-p_1)} \sim \left[(P-p_1)^2\right]^{N-3} \overset{\Downarrow}{=} \left[E^2 - 2EE_1\right]^{N-3}$$

$$\sim \left(1 - \frac{E_1}{E/2N}\,\frac{N-3}{N}\,\frac{1}{N-3}\right)^{N-3}$$

$$\underset{N\gg 1}{\sim} \boxed{\exp\left(-\frac{E_1}{E/2N}\right)}$$

exponential in E_1 !!

q.e.d.

$$\Rightarrow \boxed{E_1 \frac{d\sigma}{d^3\vec{p}_1} = C(E,N)\, e^{-\frac{E_1}{E/2N}}\ \Delta(E, p_1)}$$

If there is no non-trivial ("interesting") dynamics, i.e., $\Delta(E,p_1) \equiv \Delta(E)$, m_T-spectrum looks exactly like in thermal equilibrium with temperature

$$\boxed{T = \frac{1}{2}\frac{E}{N}} \quad !$$

Unfortunately, an ultrarelativistic gas in thermal equilibrium has

$$\boxed{T = \frac{1}{3}\frac{E}{N}} \quad !$$

Note:

$$\Phi(P,N) \equiv \int \prod_{i=1}^{N} \frac{d^3\vec{p}_i}{E_i}\, \delta^{(4)}\left(P - \sum_{i=1}^{N} p_i\right)$$

is not a microcanonical partition function,

$$\Omega(P,N) \sim V^N \int \prod_{i=1}^{N} d^3\vec{p}_i\, \delta^{(4)}\left(P - \sum_{i=1}^{N} p_i\right)$$

$$= \int \prod_{i=1}^{N} \left(\frac{d^3\vec{p}_i}{E_i}\, p_i^{\mu} V_{\mu}\right) \delta^{(4)}\left(P - \sum_{i=1}^{N} p_i\right) \quad !$$

R. Hagedorn, J. Rafelski, PLB 97, 136 (80)

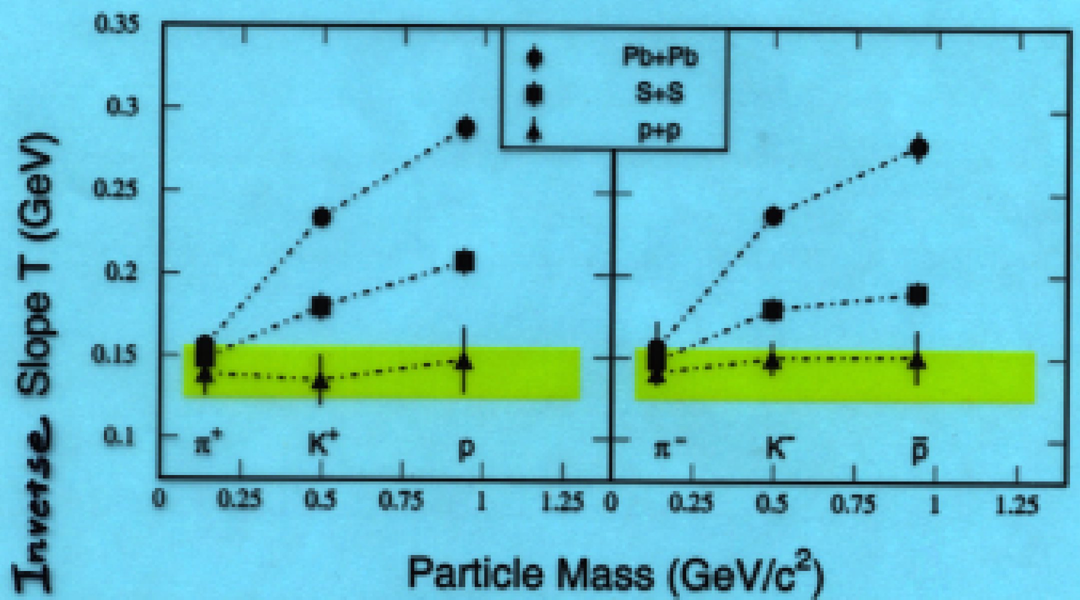
However: One can consider parts of phase space, and Laplace-transform w.r.t. E and N:

$$\Phi(\beta,\lambda) \equiv \sum_{N} \lambda^N \int_0^\infty dE\, e^{-\beta E}\, \Phi(E,N)$$

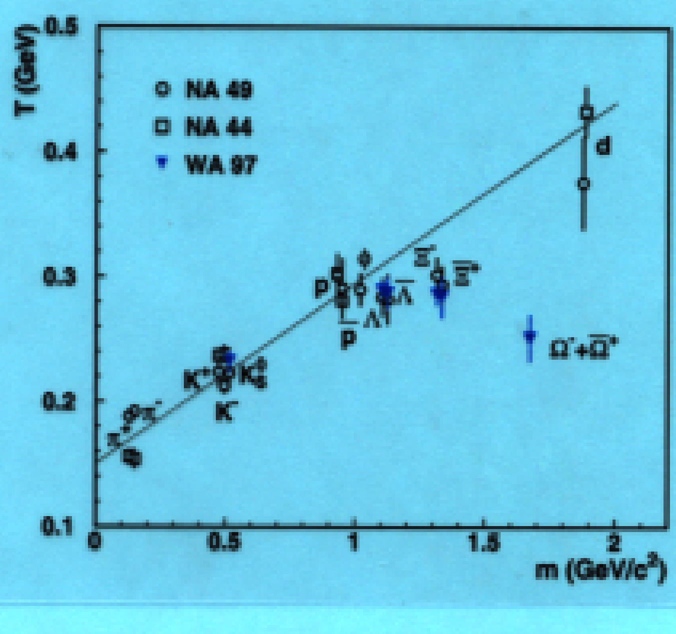
The "fugacity" λ is like an ordinary thermodynamic fugacity !

Inverse slope characteristics:

CERN-SPS energies:



NA 44 coll.
I.G. Bearden et al.,
PRL 78, 2080 (97)



P. Seyboth,
CERN seminar,
7-11-2000
http://na49info.cern.ch/na49/Archives/Images/Talks/Seyboth_cern00

$\Rightarrow$ Inverse slope $= a + b\,m$;

$a \simeq 150$ MeV, does not depend on A ;

b increases with A.

Inverse slope characteristics suggest:

there is collective motion!

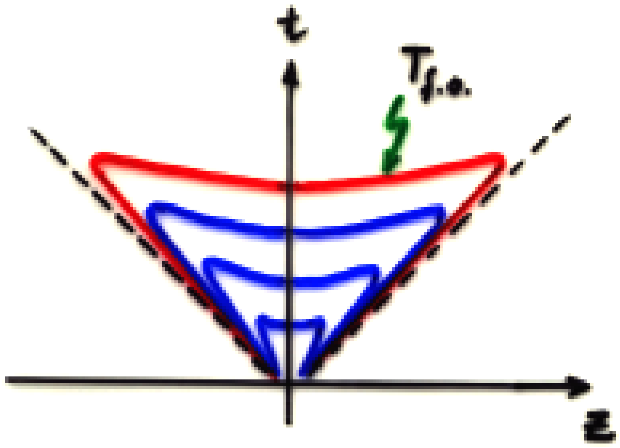
(**Example:** Expanding, thermal, non-rel. source:

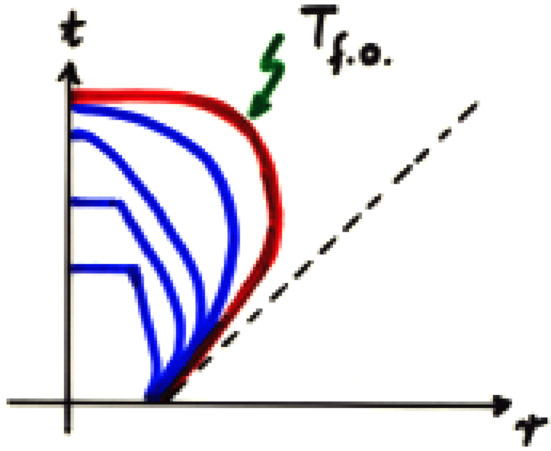
Inverse slope $\sim \frac{E}{N} = \underbrace{\frac{3}{2} T}_{a} + \underbrace{\frac{1}{2} \langle u_T^2 \rangle m}_{b}$)

$\Rightarrow$ If there is collective motion, there must have been multiple interactions of produced particles!

$\Rightarrow$ Multiple interactions of produced particles (eventually) lead to thermalization!

Local thermalization ⇒ Ideal fluid dynamics





Particles decouple ("freeze out") from fluid evolution at temperature $T_{f.o.}$,

$$\boxed{E\frac{dN}{d^3\vec{p}} = \int d^4x\, S(x,p)\,, \quad S(x,p) = \int_{\Sigma_{f.o.}} p\cdot d\Sigma\, f(x,p)\,\delta^{(4)}(x-\Sigma_{f.o.})}$$

source function

F. Cooper, G. Frye, PRD 10, 186 (74)

Pb+Pb, E_{Lab} = 158 AGeV @ CERN-SPS:

	$T_{f.o.}$ [MeV]	$\langle v_T \rangle$
B.R. Schlei, Heavy Ion Physics 5, 403 (97)	140	0.55
C.M. Hung, E. Shuryak, PRC 57, 1891 (98)	110–120	0.6
P. Huovinen, P.V. Ruuskanen, J. Sollfrank, NPA 650, 227 (99)	120–140	
A. Dumitru, D.H.R., PRC 59, 354 (99)	130	0.5
B. Kämpfer, hep-ph/9612336	120	0.43
B. Tomášik, U.A. Wiedemann, U. Heinz, nucl-th/9907096	100	0.55
A. Ster, T. Csörgő, B. Lörstad, NPA 661, 419c (99)	140	0.55

Particle ratios at freeze-out:

(1) Assume T and μ_i are constant on $\Sigma_{f.o.}$.

(2) Assume all particle species freeze out on the same $\Sigma_{f.o.}$ with the same u^μ.

$$\Rightarrow \quad \boxed{\frac{N_i}{N_j} = \frac{n_i(T,\mu_i)}{n_j(T,\mu_j)}}$$

density of particle species i at temp. T and chemical pot. μ_i

Proof: see J. Cleymans, K. Redlich, PRC 60, 054908 (99)

(3) Assume chemical equilibrium on $\Sigma_{f.o.}$:

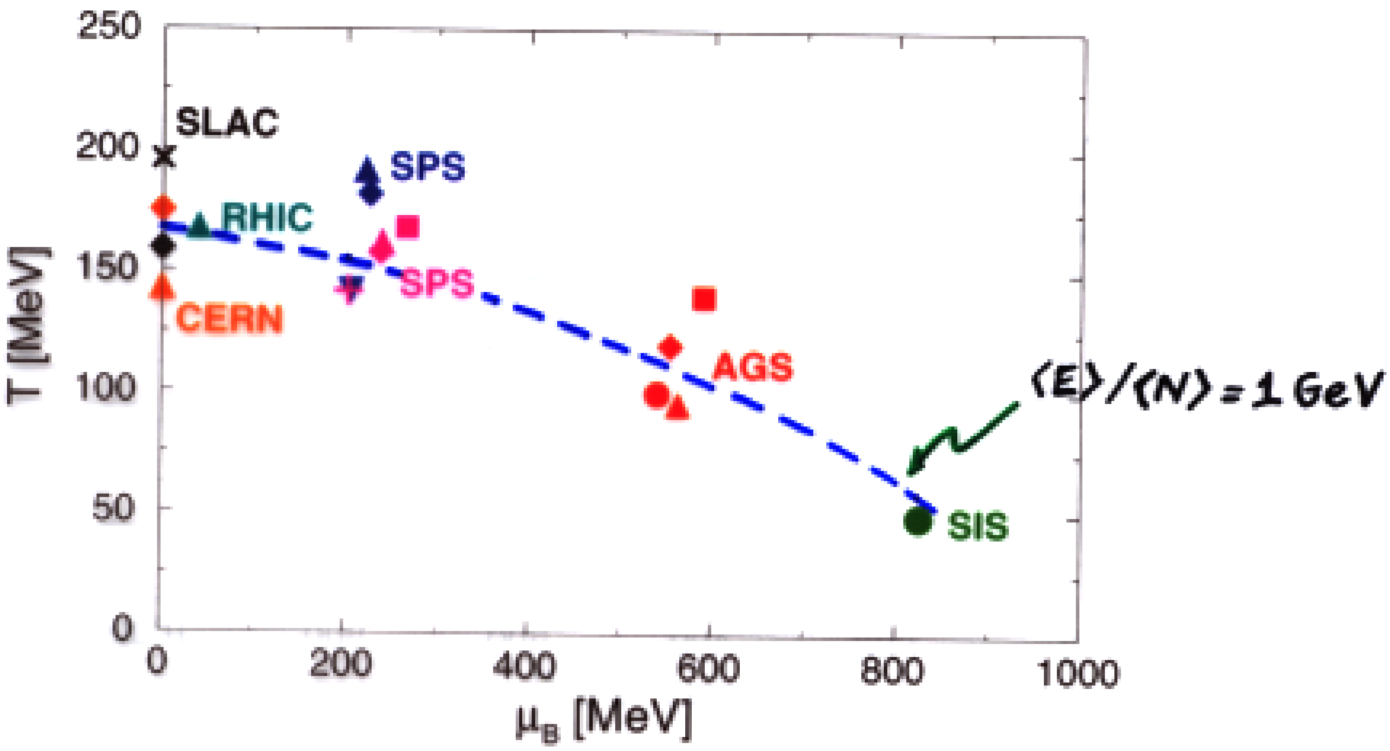
$$\Rightarrow \quad \boxed{\mu_i = b_i \mu_B + s_i \mu_S + q_i \mu_e + \dots}$$

$\lambda_i = e^{\mu_i/T}$

can be eliminated through conditions of global strangeness, electric charge, ... neutrality

$\Rightarrow$ Given hadronic masses m_i and partial decay widths $\Gamma_{i \to jk}$, all particle ratios are functions of T and μ_B only!

T, μ_B at freeze-out from particle ratios:

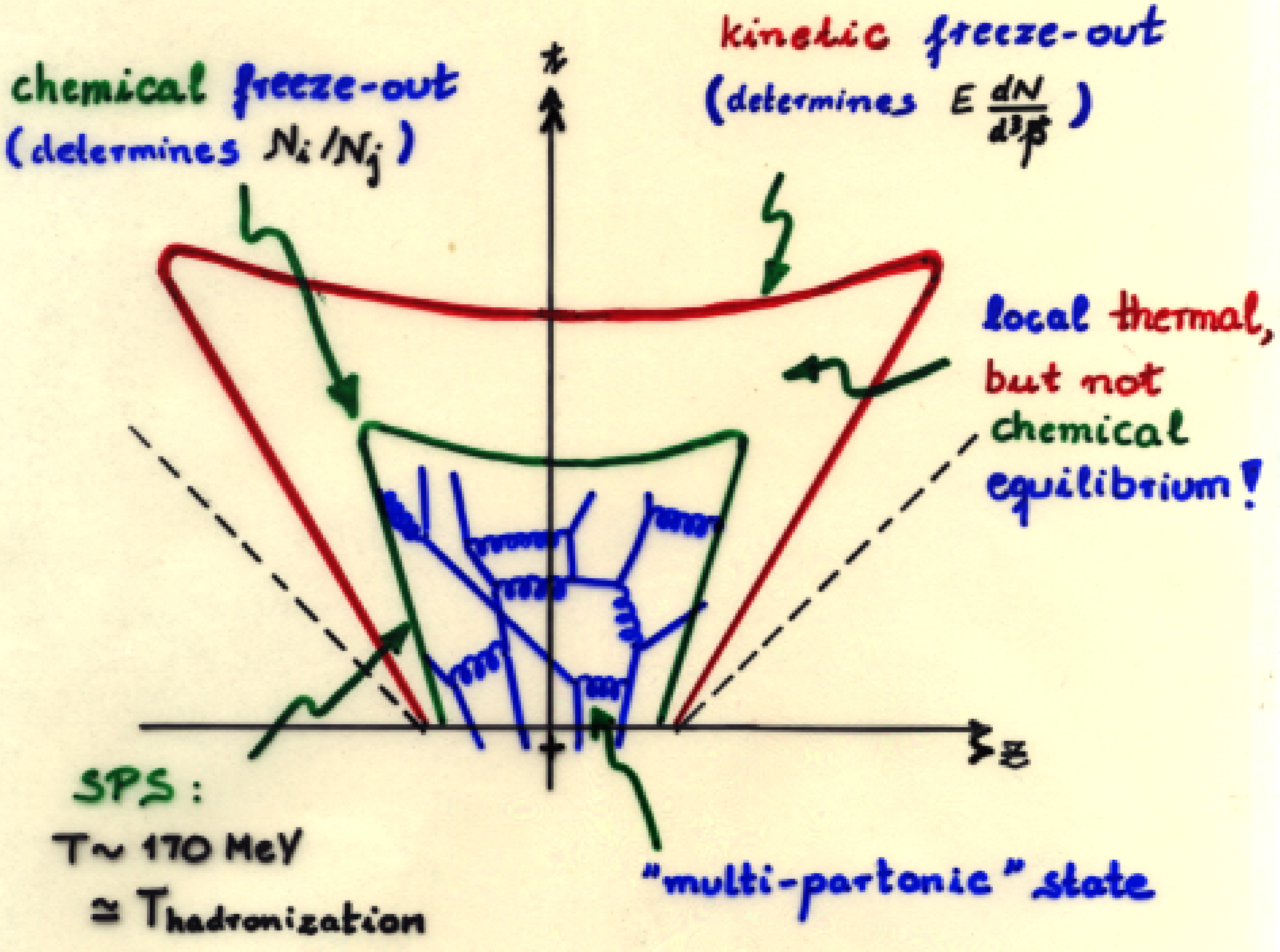


Au+Au, 2.3 AGeV	● [1] Cleymans et al., PRC 59 (99) 1663	← Exact charge conservation
	■ [2] Yen et al., JPG 24 (98) 1777	← excl. volume
Au+Au, 5 AGeV	◆ [3] Becattini et al., hep-ph/0002267	← $\gamma_s < 1$
	▲ [4] Kabana et al., hep-ph/0010247	← $\gamma_s = 1$
	● [5] Cleymans et al., PLB 388 (96) 5	
S+S, 20 AGeV	◆ [6] Becattini, JPG 23 (97) 1933	
	▼ [7] Sollfrank, EPJC 9 (9) 159	← hydro
	▲ [8] Panagiotou et al., PRC 53 (96) 1353	
	■ [9] Braun-Munzinger et al., PLB 465 (9) 15	
Pb+Pb, 17 AGeV	◆ [3]	
	▲ [4]	
	+ [10] Letessier et al., IJMPE 9 (00) 107	← $\gamma_s > 1$
Au+Au, 130 AGeV	▲ [4]	
p-pbar, 900GeV	◆ [11] Becattini et al., ZPC 74 (97) 319	
	▲ [4]	
e^+e^-, 29GeV	× [12] Hoang et al., ZPC 38 (88) 603	← fit to $E\frac{d\sigma}{d^3p}$
	◆ [13] Becattini et al., hep-ph/0010221	

cf. also: J. Sollfrank, J. Phys. G 23 (97) 1903

J. Cleymans, K. Redlich, PRC 60, 054908 (99)

$$\Rightarrow \quad \boxed{T_{f.o.}\,(\text{particle ratios}) \gtrsim T_{f.o.}\,(m_T\text{-spectra})}$$



"Primordial hadrosynthesis"

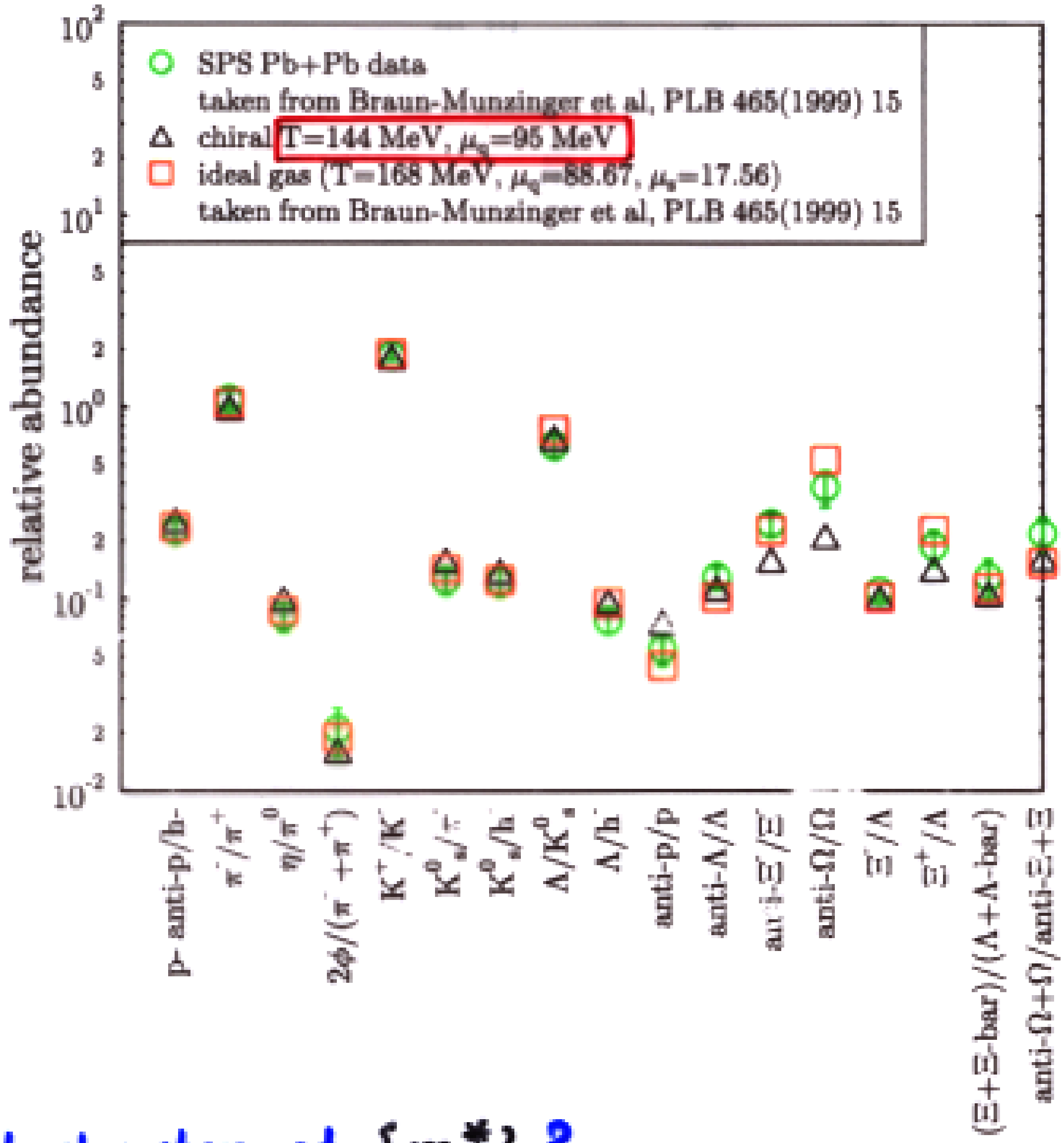
U. Heinz, NPA 661, 140c (99)
R. Stock, NPA 630, 535c (98)
PLB 456, 277 (99)

Chemical freeze-out parameters depend on $\{m_i\}$:

Replace $\{m_i\} \rightarrow \{m_i^*\}$ — medium-modified masses (effect confirmed by CERES!)

D. Zschiesche et al., nucl-th/0007033

(cf. also G.E. Brown, M. Rho, C. Song, nucl-th/0010008
W. Florkowski, W. Broniowski, PL B 477, 73 (2000))



... and why stop at $\{m_i^*\}$?

How about $\{\Gamma_{i\to jk}\} \rightarrow \{\Gamma^*_{i\to jk}\}$

Remarks (Conclusions):

(1) $\boxed{\text{A hadronic system in thermodynamical equilibrium can – by definition – not know about its past (i.e., whether it originated from a QGP)!}}$

⇒ Thermal γ's (with $T > T_{had.}$) could prove existence of QGP! (→ Theory & Experiment!)

(2) Chemical freeze-out parameters depend linearly on hadronic masses, but only logarithmically on particle ratios:

$$\frac{N_i}{N_j} \simeq \exp\left[\frac{\mu_i - \mu_j}{T} - \frac{m_i - m_j}{T}\right]$$

⇒ Need better understanding of in-medium properties of hadrons! (→ Theory!)

(3) What appears to be thermal and chemical equilibrium could be simply phase space!

⇒ Need pp and pA data! (→ Experiment!)

(To learn about dynamics, too!)

and ⇒ Need to find other observables which are indicative of thermalization!