The Free Energy of hot QCD

talk by York Schröder (Helsinki)

Quark Matter 2001, Stony Brook / BNL, 01/16/01

$$p \overset{V\to\infty}{=} -f = \frac{T}{V} \ln \int \mathcal{D}[A^a_\mu, \bar\psi_f, \psi_f]\, e^{-\int_0^{1/T} d\tau \int d^3x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar\psi_f \not{D} \psi_f\right)}$$

Contents

- State of the art
- The road to success
  [with K. Kajantie / M. Laine / K. Rummukainen, PRL 86 (2001) 10]

Ⓐ 4D lattice MC

[Boyd et. al., Karsch et. al., Okamoto et. al., ...]

extrapolate: $V \to \infty$, $a \to 0$

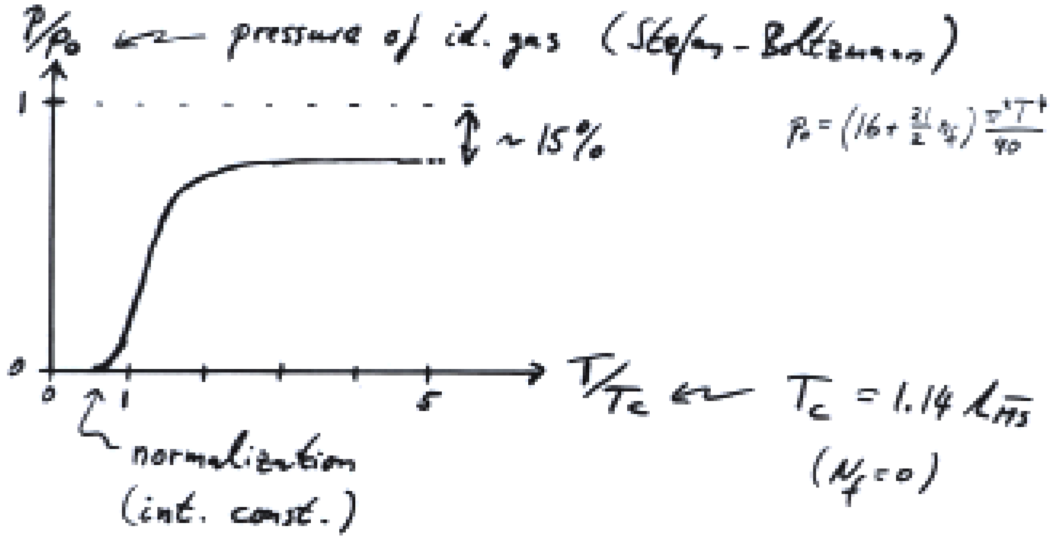
in practice, works only for $T \lesssim$ a few T_c

(technicality: $\langle \mathbb{1} \rangle \sim f$ not directly comp.

but measure $\langle S \rangle \sim \partial_\beta f$ for some β's,

integrate back to get f – mod const.)

lattice results (qual.):



$N_f = 0$ well under control

$N_f > 0$ under (rapid) development

(B) 4D pert. calc.

expand in coupling $g(T)$

$$p = p_0 \left[1 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4) g^4 + c_5 g^5 + \mathcal{O}(g^6 \ln g, g^6) \right]$$

etc.

ideal gas

[Shuryak '78]

[Kapusta '79]

[Toimela '83]

[Arnold/Zhai '94]

[Kastening/Zhai '95]

not (yet) computed

°LINDE°

c_i's known analytically! impressive work...

QCD $\to$ $SU(N_c)$ with N_f massless fermions in fund. rep. ($T \gg m_q$)

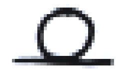
$\left(\!\left(d_A = N_c^2 - 1,\ C_A = N_c,\ d_F = N_c N_f \right.\right.$

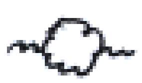
$\left.\left. \Rightarrow \text{get } U(1) \text{ via } d_A = 1,\ C_A = 0,\ d_F = N_f \right)\!\right)$

structure:

series is <u>non</u>analytic in g^2 !

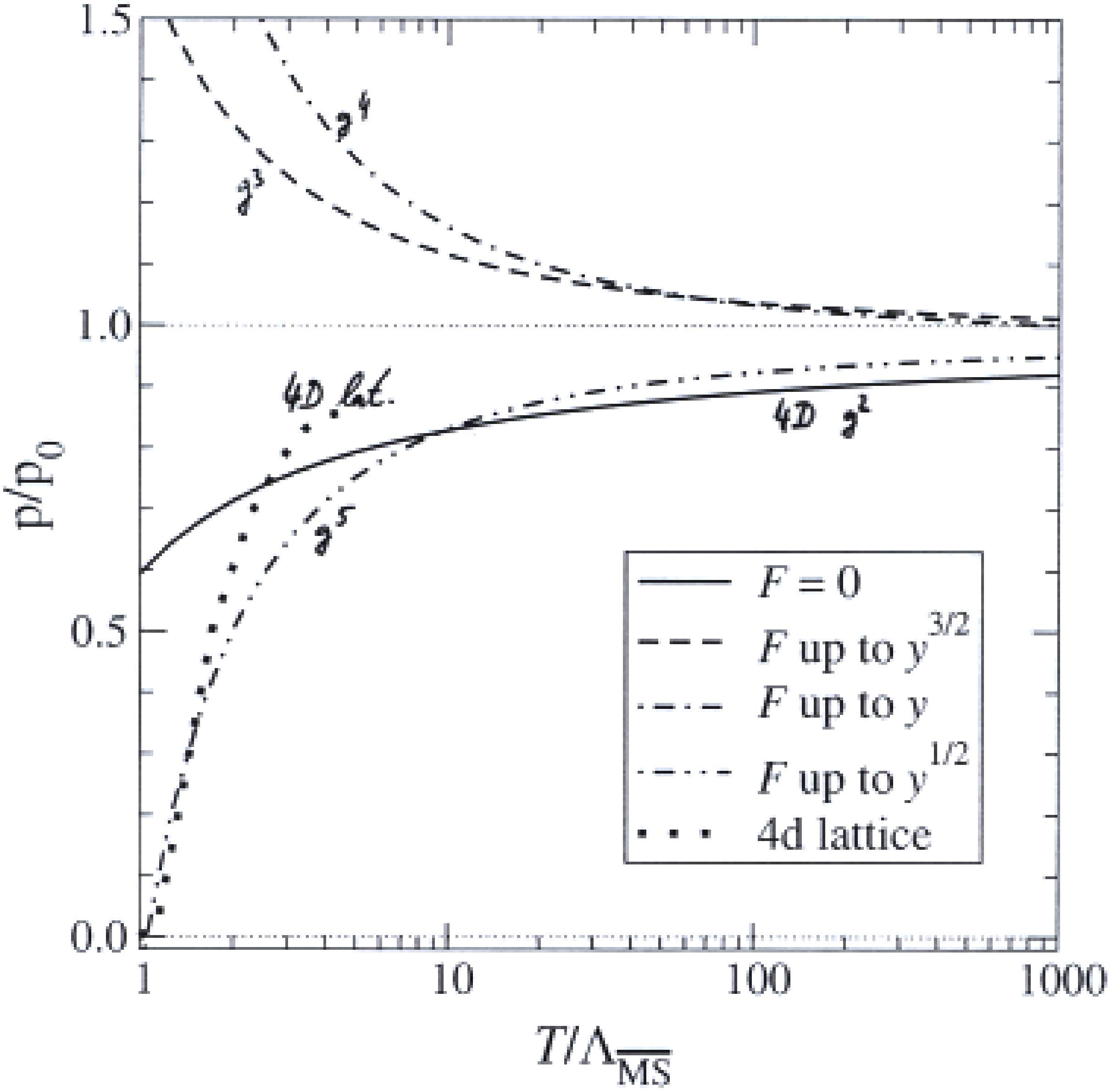
$\leftrightarrow$ Debye screening 'mass' $\leftrightarrow$ HTL resu!

(ϕ^4:  , $m^2 = \frac{1}{2} g^2 \underbrace{\sum\!\!\!\!\!\!\int_P \frac{1}{P^2}}_{= T^2/12}$, $\frac{1}{P^2} \to \frac{1}{P^2 + m^2}$)

(YM:  etc , $\Pi^{(1)}(P)$, $\frac{1}{P^2 + \Pi(P)}$, $\Pi_L^{(1)}(p_0, \vec{p}) \neq 0$

$\Pi_T^{(1)}(p_0 \to 0, \vec{p}) \to 0$)

expand in $m/T \sim g$, $\ln m \to \ln g$ (IR)



1.5
1.0
0.5
0.0
1
10
100
1000
p/p_0
T/Λ_MS
F = 0
F up to y^3/2
F up to y
F up to y^1/2
4d lattice
g^4
g^3
4D lat.
4D g^2
g^5

(C) resummation

[Andersen, Braaten, ... '99 '00]
[Blaizot, Iancu, Rebhan '99 '00]
[Peshier '99] [Parwani '00]

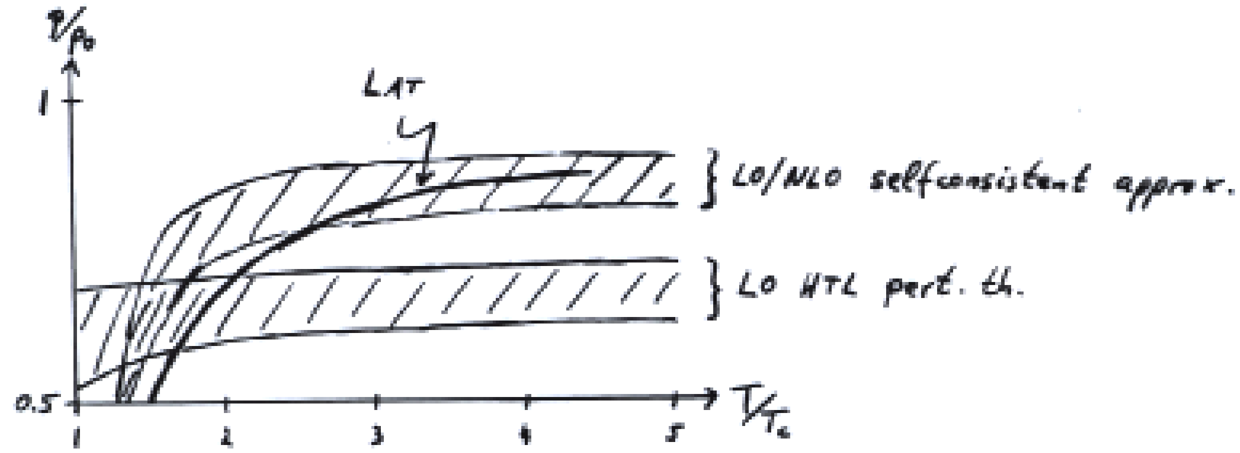
use effective masses, tadpole improvements, ...

But neglect / suppress IR (long-distance) effects

$\rightsquigarrow$ work only for short-distance dominated observables.

NB in the case of f, there is an intricate cancellation of long-dist. effects. $\rightsquigarrow$ see next sect's

But one needs a priori knowledge ...



$\boxed{\text{(D) dimensional reduction}}$

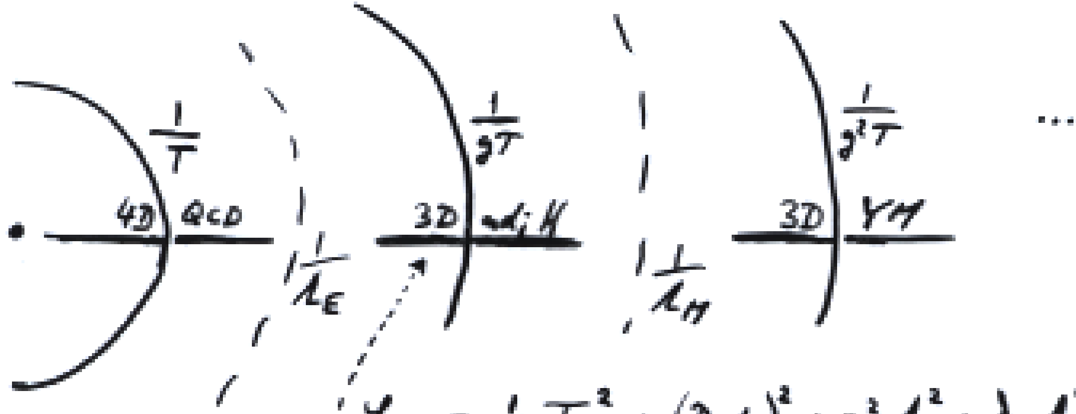
[Appelquist, Pisarski '81]
[Nadkarni '88]
[Huang, Lissia '95]
[Kajantie et.al. '96], ...

for $\frac{g^2}{4\pi} \ll 1$ (i.e. $T \gg T_c$)

have scale hierarchy $T, gT, \ldots$

$\rightsquigarrow$ construct effective theories ('integrate out heavy modes')

distance scales:



$$\mathcal{L}_{3D} = \frac{1}{4} F_{ij}^2 + (D_i A_0)^2 + m_D^2 A_0^2 + \lambda_A A_0^4$$

small coupling allows perturbative reduction

$$\frac{g_3^2}{T} = \frac{8\pi^2}{11 \ln(6.742\, T/\Lambda_{\overline{MS}})} \qquad (NLO,\ FAC,\ N_f = 0,\ N = 3)$$

$$x = \frac{\lambda_A}{g_3^2} = \frac{3}{11 \ln(5.371\, T/\Lambda_{\overline{MS}})}$$

$$y = \frac{m_D^2}{g_3^4} = \frac{3}{8\pi^2 x} + \frac{9}{16\pi^2} + \mathcal{O}(x)$$

BUT 3D adj H is confining $\to$ nonperturbative.

$\rightsquigarrow$ split $f = f_E(\text{scale } T) + f_M(\text{3D eff. th.})$

[Braaten / Nieto '96]

f_E: perturbative, short-distance

f_M: non-part., long-distance

up to hard-mode $\mathcal{O}(g^6)$ terms,

$$\frac{p(T)}{p_0(T)} = 1 - \frac{5}{2}x - \frac{45}{8\pi^2}\left(\frac{g_3^2}{T}\right)^3\left(\mathcal{F}_{\overline{MS}}(x,y) - 24\frac{4}{(4\pi)^4}\left[\ln\frac{\bar\mu_3}{T} + \underbrace{\delta}_{=1.35\times10^{-4}}\right]\right)$$

$$= -\frac{1}{V g_3^6}\ln\int\mathcal{D}[A_i^a, A_0^a]\, e^{-\int d^3x\, \mathcal{L}_{3D}}$$

(pert.) $= d_3\, y^{3/2} + \left(d_4'\left(\frac{1}{2}\ln y - \ln\frac{\bar\mu_3}{g_3^2}\right) + d_4\right)y + \left(d_5'\ln y + d_5\right)y^{1/2}$

① etc.

$+\, \Delta\mathcal{F}_{\overline{MS}}(x,y)$

collects all higher orders

- factorization scale $\bar\mu_3$ cancels in pressure
- this computation is equivalent to hard 4D 3loop
- achieved clean separation of contrib's
- convergence (still) bad (even for $x \neq 0$)

$\rightsquigarrow$ not (yet) gotten further, but understood structure (multi-scale). LINDE in $\Delta\tilde{\mathcal{F}}_{\overline{MS}}$!

the 'road'

'measure' $\mathcal{F}_{\overline{MS}}$ via 3D MC instead!

again, take derivatives: $\partial_y \mathcal{F}_{\overline{MS}} = \langle A_0^2 \rangle_{\overline{MS}}$

$\partial_x \mathcal{F}_{\overline{MS}} = \langle A_0^4 \rangle_{\overline{MS}} + \dots$

since $y(x)$

integrate back $\mathcal{F}_{\overline{MS}} = \mathcal{F}_{\overline{MS}}(y_0) + \int_{y_0}^{y} dy \left(\partial_y \mathcal{F}_{\overline{MS}} + \frac{dx}{dy} \partial_x \mathcal{F}_{\overline{MS}} \right)$

choose y_0 to get int. const perturbatively

e.g. at $T_0 = 10^{11} \Lambda_{\overline{MS}} \hat{=} y_0 = 3.86$

now real hard work is hidden in details:

- relate L, $\overline{MS}$ schemes
  → compute diags in lat. reg. scheme 3loop ✓✓
- need $(a \to 0)$ divergences of condensates
  → 3loop 2pt fcts / 4loop vac in A_0 sector
- accuracy of $\mathcal{F}_{\overline{MS}}(x_0, y_0)$ crucial!
  → improve with coeff. $(d_6' \log y) y^0$
- MC + numeric back-integration involved also ...
- control of higher order operators in $\mathcal{L}_{eff}$:
  → assure that $\langle A_0^4 \rangle$-contrib. small.

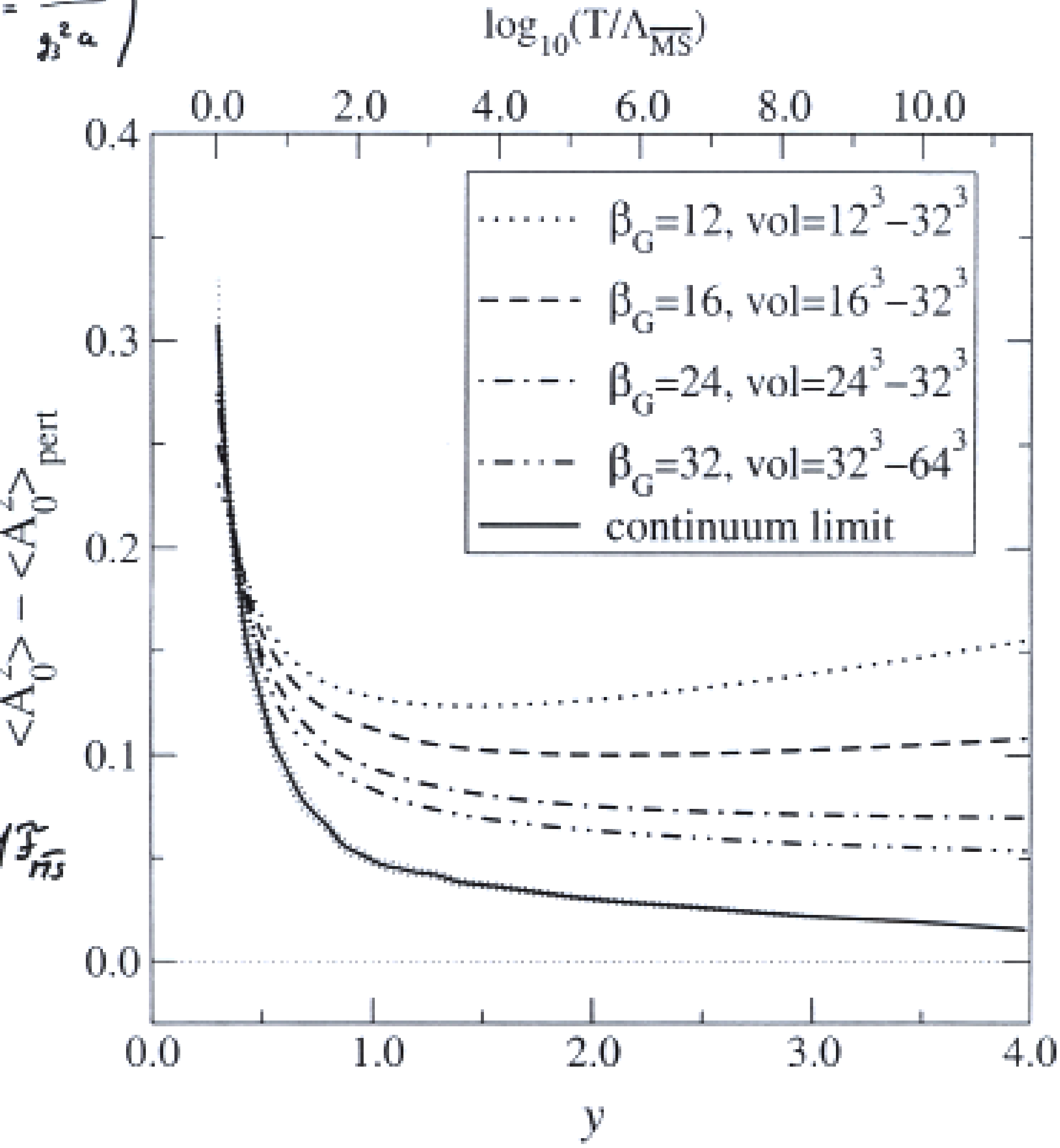
how far we are now:

$\langle A_0^2 \rangle$ measured ✓ → large non-pert. contrib. for $T \to T_c$

4 loop cont/lat $\mathcal{Y}$ → allow for int. const. e_0

from $\Delta \mathcal{F}_{\overline{MS}}(x_0, y_0) = e_0 \frac{y^0}{(4\pi)^4} d_A C_A^3 (1 + \mathcal{O}(x_0))$

$\left(\beta_G = \frac{6}{g_3^2 a} \right)$



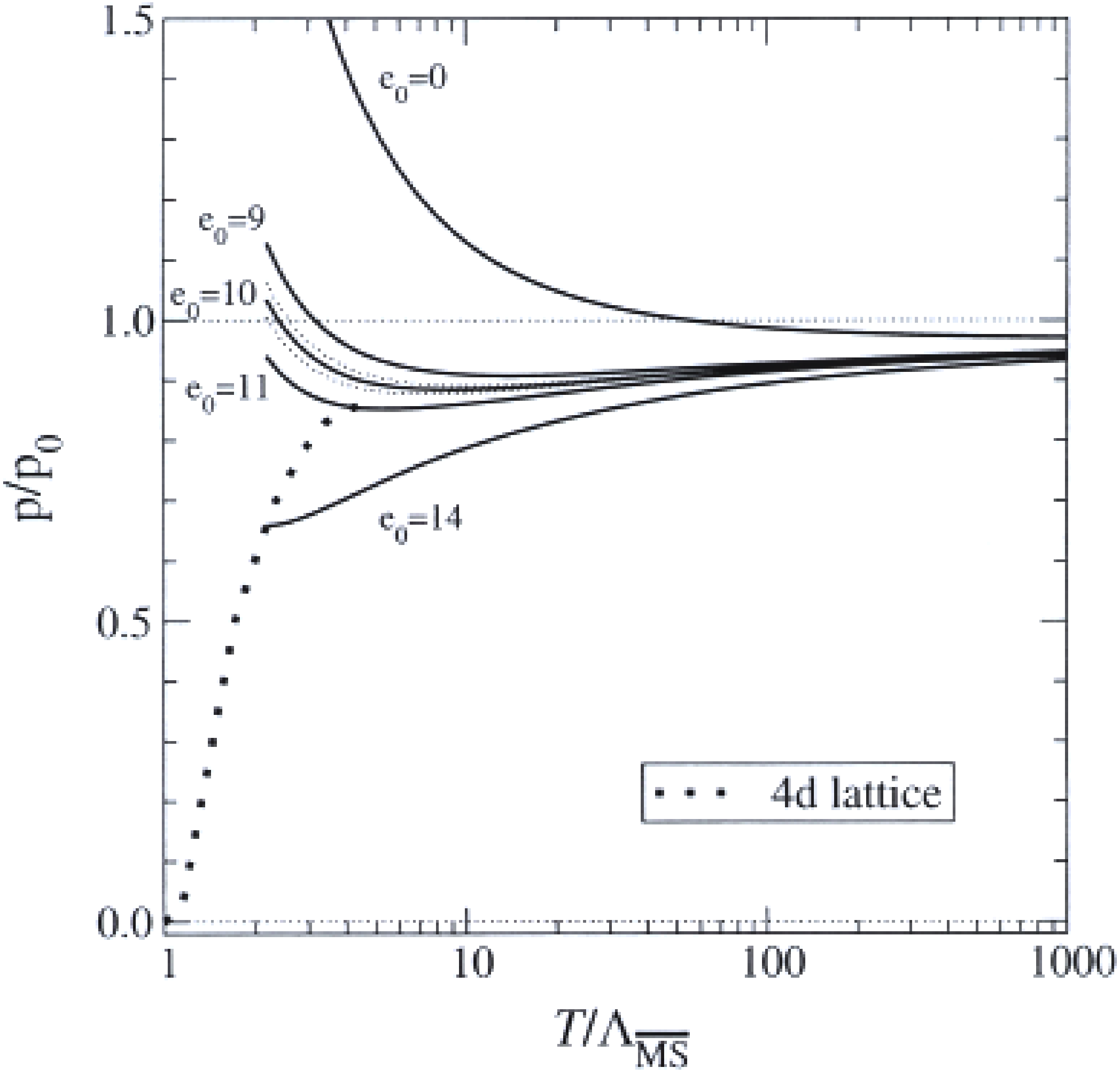
$g_3 \Delta \mathcal{F}_{\overline{MS}}$

status:

4D hard modes (3 loop)

+ 3D lattice MC ($\langle \lambda_0^2 \rangle$ only)

+ free param. in int. const. (e_0)



(observe cancellation of long-dist effects @ $T \gtrsim 30 \Lambda_{\overline{MS}}$)

Conclusions

- F is known poorly over a huge T-interval.

  4D lattice: up to $\sim 5T_c$

  pert. theory: down from $T=\infty$ to ..?
- effective theory separates scales,

  provides clear separation of pert and non-pert

  contrib's.
- A combined perturbative + 3D lattice MC

  seems to offer a (<u>the</u>?) solution
- Hard work to do; in progress...