COLOR SUPERCONDUCTIVITY IN COMPACT STARS

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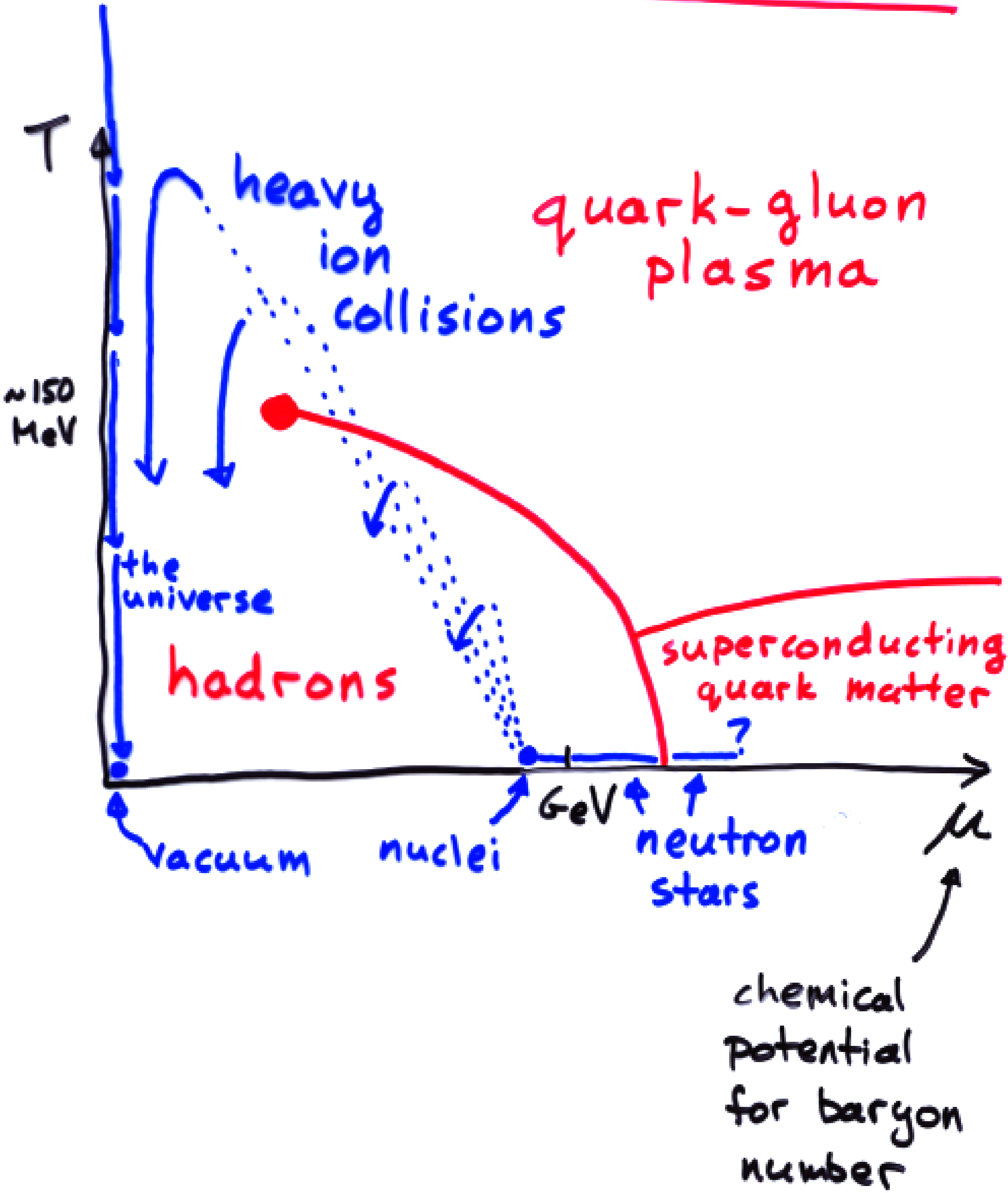
Quark Matter 2001
BNL Day

THE QCD PHASE DIAGRAM



THE "IDEAL" COLOR-FLAVOR LOCKED PHASE

Cold dense quark matter with $m_s = m_u = m_d$.

Attraction between quarks which are antisymmetric in color makes Fermi surface unstable to Cooper pairing.

Favored channel (all 9 quarks pair) has $\langle ud \rangle = \langle us \rangle = \langle ds \rangle$

Δ of order 100 MeV

T_c of order 50 MeV

color-flavor locking → broken chiral symmetry

All 9 quarks gapped.

Light degrees of freedom:

- superfluid mode (massless)
- 8 light pseudo-NGB from χSB.
- massless $\tilde{Q}$-photon; lin. comb. of $\gamma + g$.

  (8 gluons + 1 photon → 8 massive vectors + 1 $\tilde{\gamma}$-photon.)

At T=0: CFL is a TRANSPARENT INSULATOR

HOW DOES THE IDEAL CFL PHASE RESPOND TO THE "STRESSES OF REALITY"?

Ideal CFL: $m_s = m_u = m_d$ $\quad \mu_s = \mu_u = \mu_d$

$$N_s = N_u = N_d$$

$$\langle us\rangle = \langle ds\rangle = \langle ud\rangle$$

Reality: $m_s > m_{u,d}$ $\quad \mu_u = \mu - \frac{2}{3}\mu_e$

$$\mu_s = \mu_d = \mu + \frac{1}{3}\mu_e$$

(eg: $\mu_e \neq 0$ due to contact with surrounding nuclear matter)

If there were no pairing, those stresses ($m_s \neq 0$; $\mu_e \neq 0$) $\Rightarrow$ $N_s \neq N_u \neq N_d$

BUT If $N_u \neq N_d \neq N_s$ requires breaking CFL pairs, and ∴ costs energy, CFL pairing enforces $N_u = N_d = N_s$ in reality as in ideality.

To investigate this, we use ....

KR F. Wilczek

SIMPLIFIED TWO-QUARK MODEL

$$\mu_1 = \mu - \delta\mu \qquad \mu_2 = \mu + \delta\mu$$

$$m_1 = 0 \qquad m_2 = m_s$$

Suppose $N_1 = N_2$ and 1,2 pair w̄ gap Δ_0.

To split N_1, N_2, break a pair and then turn a 2 into a 1.

Cost: $2\Delta_0$ BENEFIT: $\frac{m_s^2}{2\mu} - 2\delta\mu$

(for $m_s \ll \mu$)

∴ $N_1 = N_2$ is stable if

$$\left| \frac{m_s^2}{2\mu} - 2\delta\mu \right| < 2\Delta_0$$

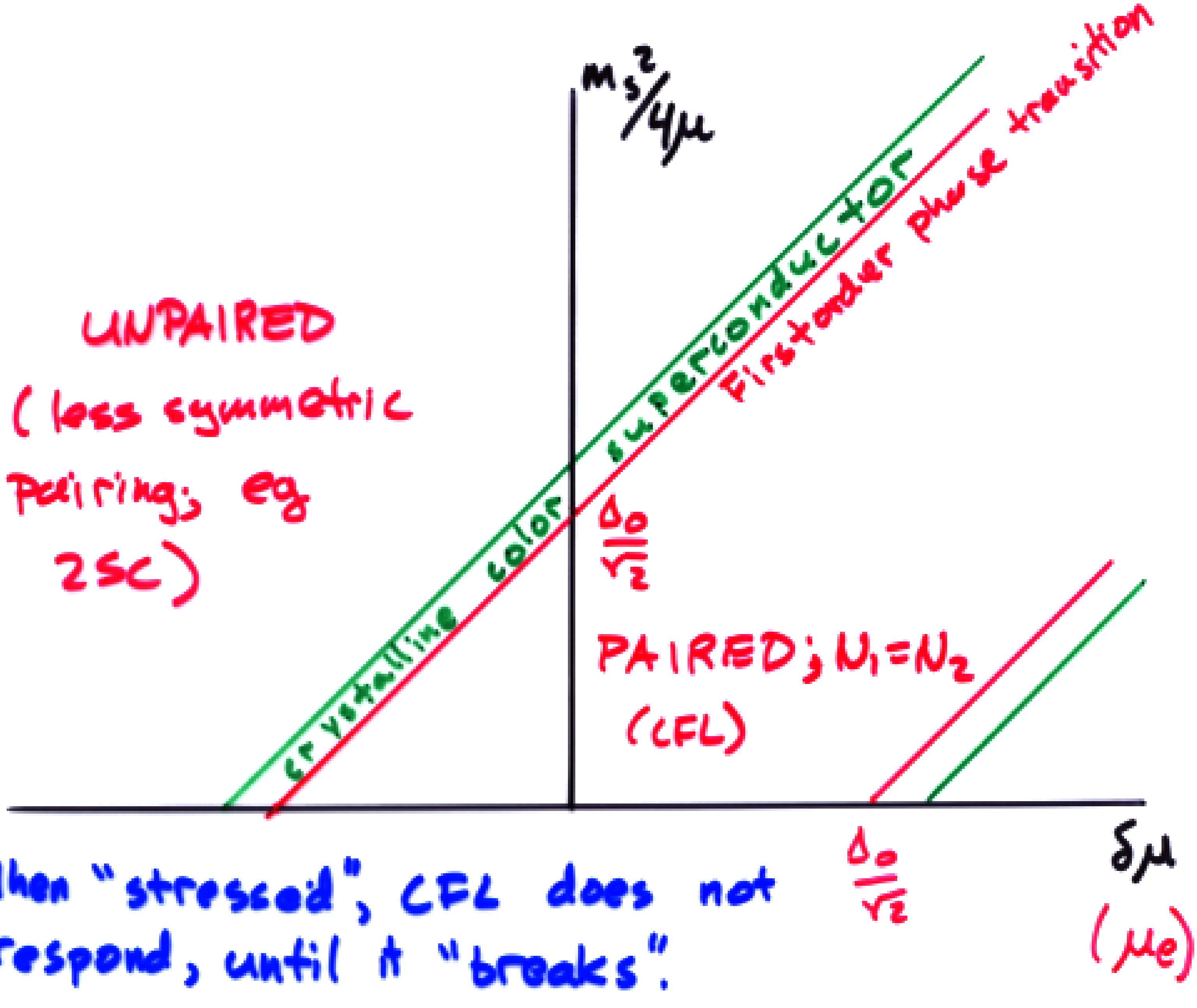
same, w̄ $1 \to 2$.

∴ BCS state, with $N_1 = N_2$; gap Δ_0 is stable, unchanged, even when $\underline{m_s \neq 0}$ $\underline{\delta\mu \neq 0}$.

BUT, WHEN IS PAIRED STATE FAVORED?

$$\underset{N_1 = N_2}{\Omega_{BCS}} - \underset{\substack{\Delta = 0 \\ N_1 \neq N_2}}{\Omega_{Normal}} = \frac{\mu^2}{\pi^2}\left[\left(\frac{M_s^2}{4\mu} - \delta\mu\right)^2 - \frac{\Delta_0^2}{2}\right]$$

$\therefore$ paired state favored for $\left|\frac{M_s^2}{4\mu} - \delta\mu\right| < \frac{\Delta_0}{\sqrt{2}}$



When "stressed", CFL does not respond, until it "breaks".

WHEREVER THE CFL PHASE IS PRESENT

i) It has $N_u = N_d = N_s$ and is electrically neutral in absence of electrons.

ii) It keeps electrons out, even if you try to push them in with a μ_e.

iii) It is a Transparent Insulator.

iv) Specific heat (small) and transport dominated by excitations of CFL quark matter itself, not by electrons.

(- superfluid mode
- pseudo-NGB from χSB
- $\tilde{Q}$-photon)

WHEREVER* CFL "BREAKS"....

- Get less symmetrically paired QM. (eg 2SC)
- Crystalline color superconducting interlude. (Certain at $M_s = 0$; $M_s \neq 0$ calculation in progress. J Kundu + KR
- * Subtleties remain. $1/\sqrt{2}$ in quantitative criterion will differ for CFL. (valid in 2-flavor toy model)

COLOR SUPERCONDUCTIVITY IN COMPACT STARS

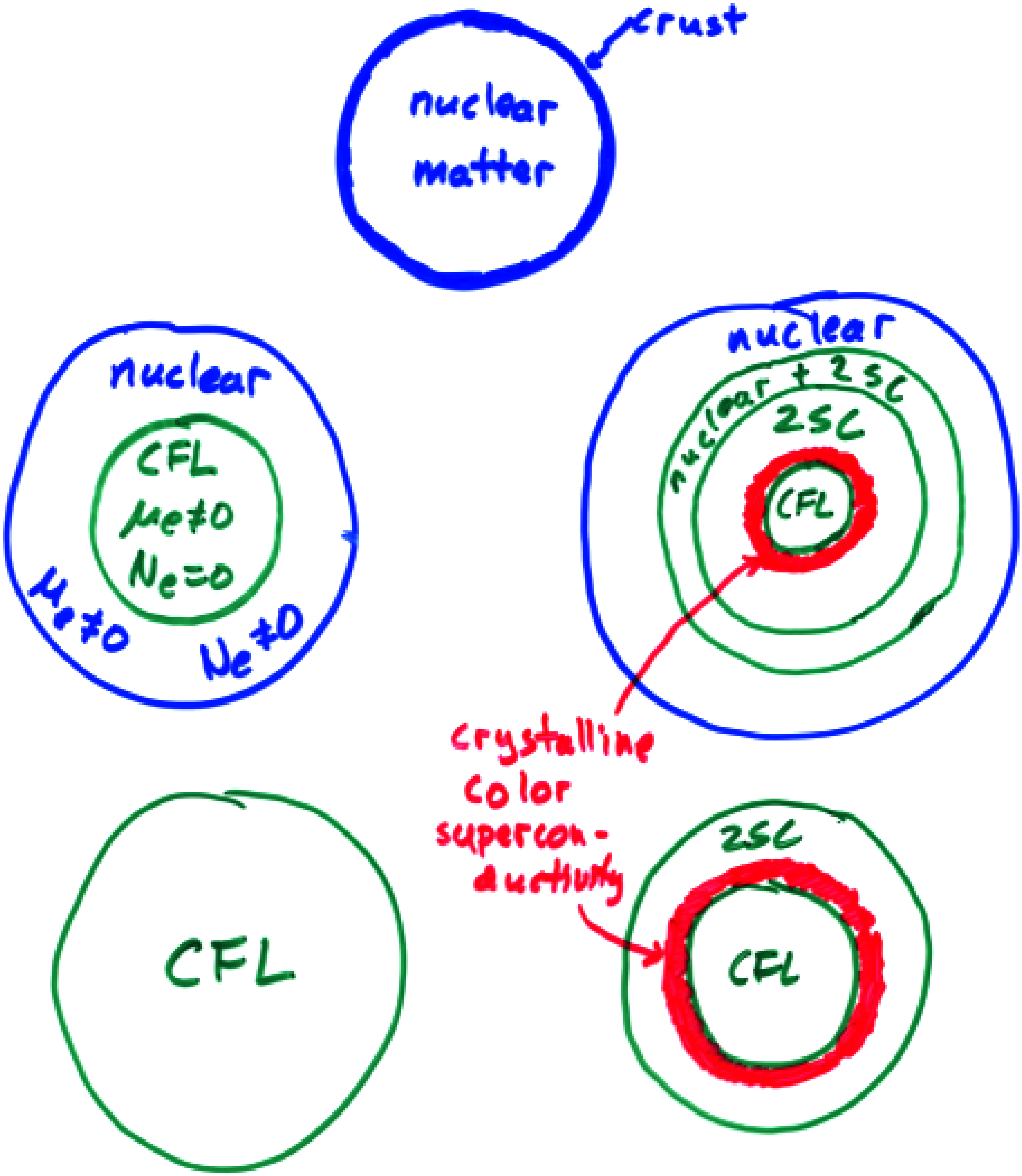
If neutron star cores are made of quark matter, it IS color superconductor. ($T_{NS} \lll T_c$)

Not clear, though, whether N.S. cores made of:

- 2SC quark matter
- CFL " "
- baryonic matter

How can we use neutron star phenomena to learn what their cores are made of?

SEVERAL SCENARIOS



Equation of State: $P(\rho)$

→ density profile

→ radius vs. mass

→ moment of inertia vs. ω (Glendenning, Weber

Presence of QM does have effects.

Color superconductivity is a Fermi surface phenomenon. Little effect on pressure. ($\sim \frac{\Delta^2}{\mu^2}$.

5-10% correction to pressure of "naive" quark matter.

BUT sharp density discontinuity is new possibility; because CFL is insulator.

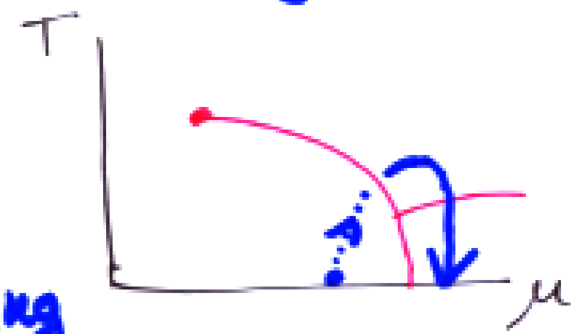
Can this give effects on GW emitted during inspiral??

Equation of State: Presence of quark matter has effects; color superconductivity has little effect.

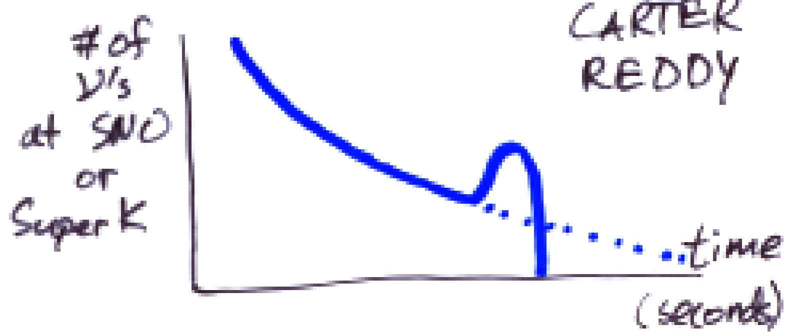
Neutron star cooling: rapid cooling by ν-emission if some quarks have $\Delta \lesssim T$. Fully gapped Q.M. (ie CFL) is inert. Ie this can teach us value of smallest gap.

Page
Prakash
Lattimer
Steiner;
Blaschke
Klahn
Voskresensky

Supernova ν's: Phase transition to c.s. during first ~10 seconds of cooling of proto-neutron-star → sudden onset of ν-transparency → sudden release of burst of ν.........
(Not yet clear if signal survives later "processing")



CARTER
REDDY



r-mode oscillation instability:

Neutron star with Ω large enough & shear, bulk viscosity small enough unstable to oscillations which reduce Ω by emission of gravitational waves.

- Quark stars made entirely of QM with gaps > 0.1 MeV (ie CFL) inconsistent (grossly) with observed Ω's. Madsen
- Alas, no constraint on neutron stars with ordinary crust + mantle, and QM core. (Friction at crust-mantle boundary prevents instability regardless of what QM core may do. Bildsten Ushomirsky)

$\vec{B}$ Evolution: Behavior of $\vec{B}$ very different in baryonic core vs. QM core (due to $\tilde{Q}$-photon, partial Meissner effect, in 2SC + CFL)

Alford
Berges
KR

BUT: not clear how observed total dipole moment is affected.

Glitches: Pairing between quarks with differing μ can lead to:

CRYSTALLINE COLOR SUPERCONDUCTOR

Alford
Bowers
KR

→ pinned rotational vortices

→ glitches originating in quark matter.

→ removes best argument against idea that pulsars could be "strange quark stars", made entirely of quark matter.

CRYSTALLINE COLOR SUPERCONDUCTIVITY

by: Alford, Bowers, KR

Bowers, Kundu, KR, Shuster

See Poster by Jeff Bowers

LOFF ANSATZ

Larkin, Ovchinnikov; Fulde, Ferrell. 1964

Considered effect of Zeeman splitting
on $\langle e_\uparrow e_\downarrow \rangle$ superconductor.
(Thinking of magnetic impurities.)

LOFF state never unambiguously seen
in condensed matter. Problem:
$\vec{B} \to$ orbital effects, not just Zeeman.

In QCD, we have JUST "Flavor Zeeman".

<u>IDEA</u>: Try Cooper pairs with total momentum!

$$(\vec{p} + \vec{q}, -\vec{p} + \vec{q})$$

Total momentum $= 2\vec{q}$ for all pairs.

$|\vec{q}| \leftarrow$ determined variationally

$\hat{q} \leftarrow$ " spontaneously.

GOAL: allow <u>both</u> u & d in pair to be on respective F.S.

$\langle LOFF | \psi(x)\psi(x) | LOFF \rangle \sim e^{i2\vec{q}\cdot\vec{x}}$

LOFF state breaks rotational & translational symmetry.

Next....

- Write $|LOFF\rangle$ ansatz.
- Vary → gap equation.
- For what $\delta\mu$ does LOFF win over BCS, Normal?

Wherever "LOFF plane wave" beats BCS & Normal, expect LOFF crystal to do better.

Eg: $\langle \psi\psi \rangle \sim \cos(2\vec{q}\cdot\vec{x})$

OR ~ cubic crystal ; or....

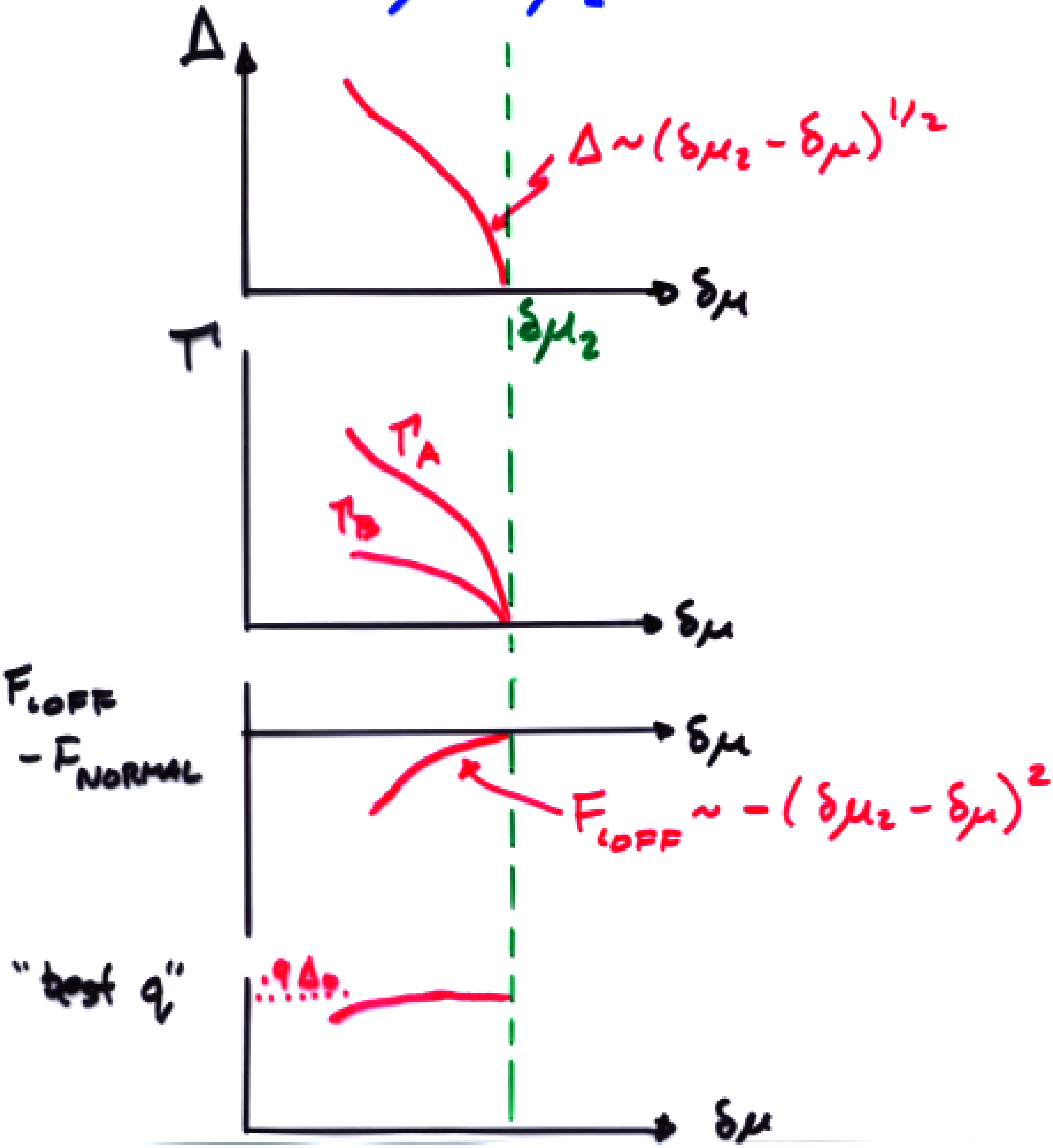
Determination of favored crystal structure

→ FUTURE WORK

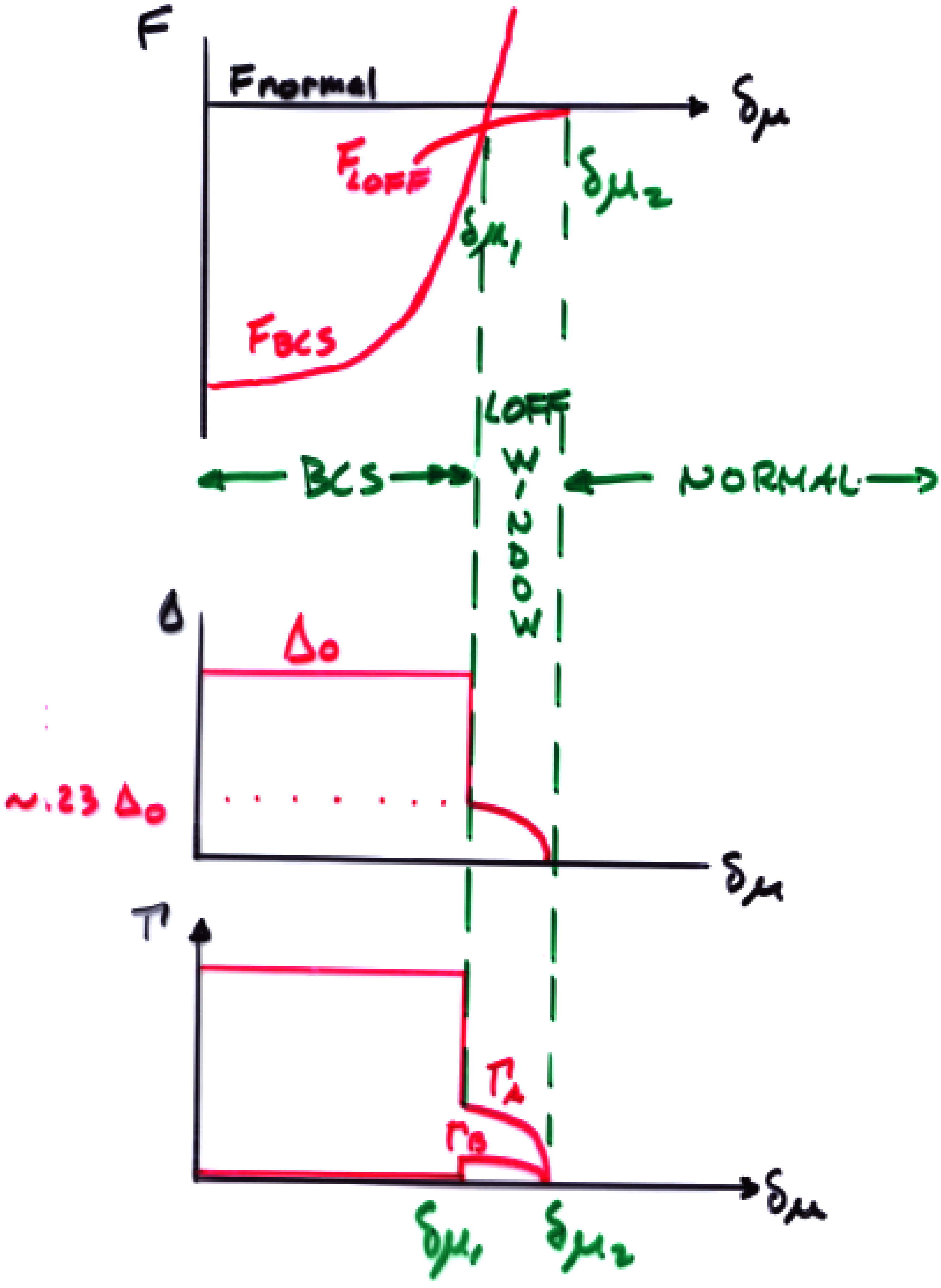
FOR WHAT $\delta\mu$ IS LOFF POSSIBLE?

Solution to gapequation exists for

$$\delta\mu < \delta\mu_2 \simeq 0.754\,\Delta_0$$



LOFF vs. BCS vs. NORMAL



THE LOFF WINDOW

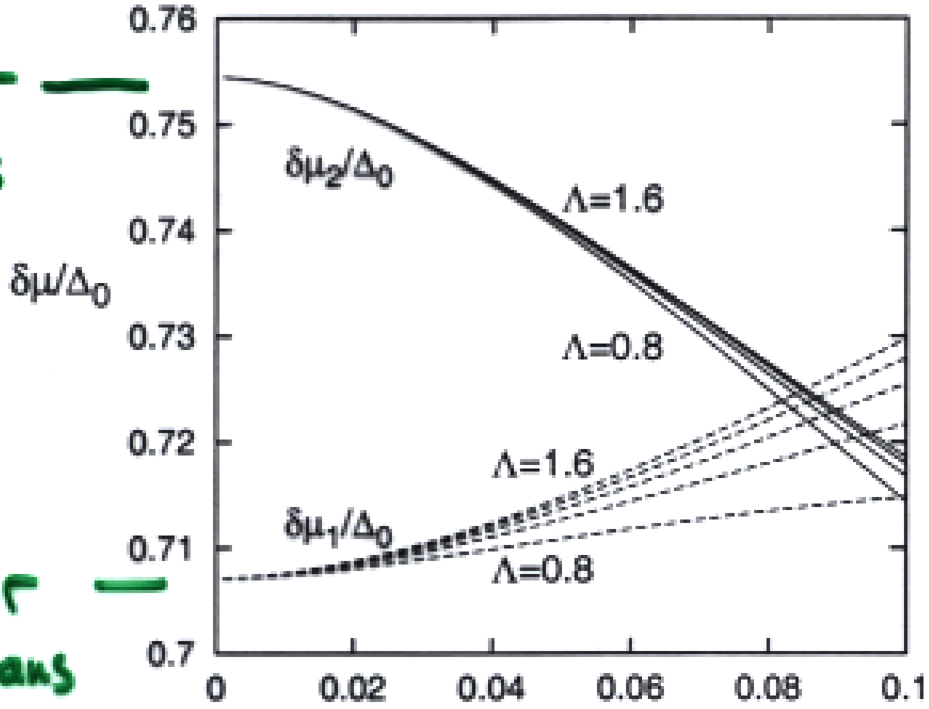
Normal

— 2^{nd} Order —
Phase Trans

LOFF

— 1^{st} Order —
Phase Trans

BCS



weak coupling $\qquad \dfrac{\Delta_0}{\text{GeV}} \qquad$ strong coupling

- stronger coupling favors BCS over LOFF
- results ~ Λ independent. (Good)

Putting in #'s (Just a game, without strange quark)

One reasonable example:

$\delta\mu \sim 30$ MeV

$\Delta_0 \sim 40$ MeV

2^{nd} ORDER P.T. @ $\delta\mu_2$: GINZBURG-LANDAU

$$\Phi(\vec{r}) \equiv \langle \epsilon_{ij}\epsilon_{\alpha\beta 3}\psi_{i\alpha}(\vec{r})C\gamma_5\psi_{j\beta}(\vec{r})\rangle$$

Normal: $\Phi = 0$ LOFF: $\Phi = \Gamma_A e^{i2\vec{q}\cdot\vec{r}}$

$$\Phi \longrightarrow \tilde{\Phi}(\vec{k})$$

$$F_{G-L} = \int d^3k \left\{ C_2(k^2)|\tilde{\Phi}(\vec{k})|^2 + C_4(k^2)|\tilde{\Phi}(\vec{k})|^4 + \ldots\ldots \right\}$$

$\delta\mu > \delta\mu_2$: $C_2 > 0$ for all k.

$\delta\mu = \delta\mu_2$: all modes stable except those with $|\vec{k}| = 2q_2$.

Our ansatz : pick one mode on this sphere spontaneously.

Reality: some crystal structure.

For one mode, $F(\Gamma_A) \sim (\delta\mu - \delta\mu_2)\Gamma_A^2 + \Gamma_A^4$

$$\Rightarrow \Gamma_A \sim (\delta\mu - \delta\mu_2)^{1/2} \qquad F \sim (\delta\mu - \delta\mu_2)^2$$

CRYSTALLINE COLOR SUPERCONDUCTIVITY IN COMPACT (NEUTRON) STARS

1. If core of neutron star is quark matter, then it IS a color supercond.

$$T_{star} \sim \text{keV} \lll T_c$$

2. Not known whether neutron stars have quark matter cores.
3. GOAL: understand observational consequences, so we can find out.
4. I will not describe the half-dozen ideas various groups are studying. Focus today is LOFF. → GLITCHES?
5. As a function of increasing depth,

μ rises

$\delta\mu$ decreases

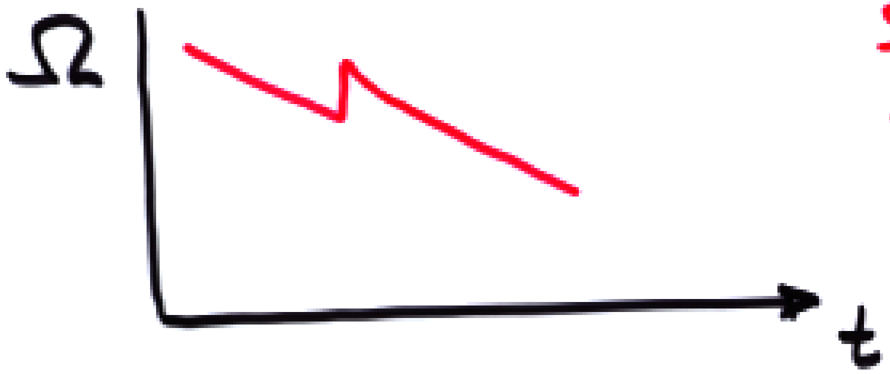
Δ changes. (rises?)

Numbers I gave before ~reasonable.

$\Rightarrow$ could have $\frac{\delta\mu}{\Delta} \in$ LOFF WINDOW in a shell.

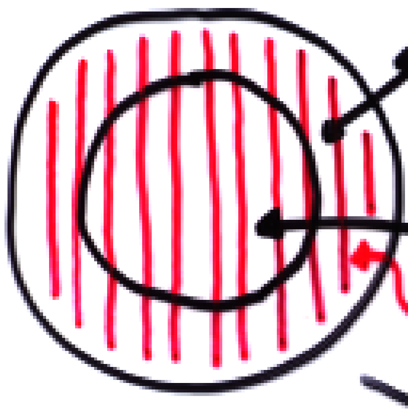
GLITCHES

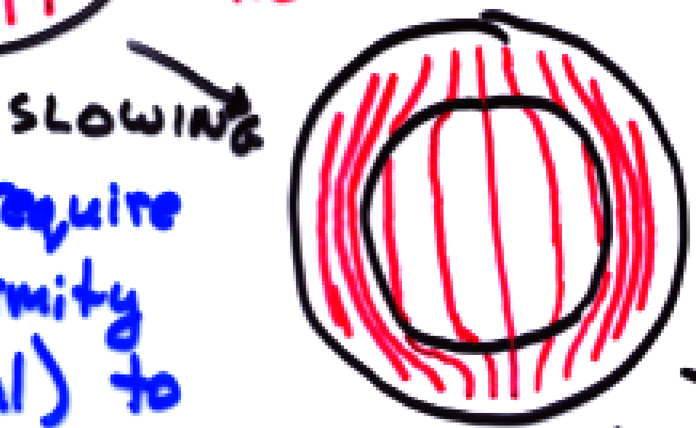
Pulsars glitch:

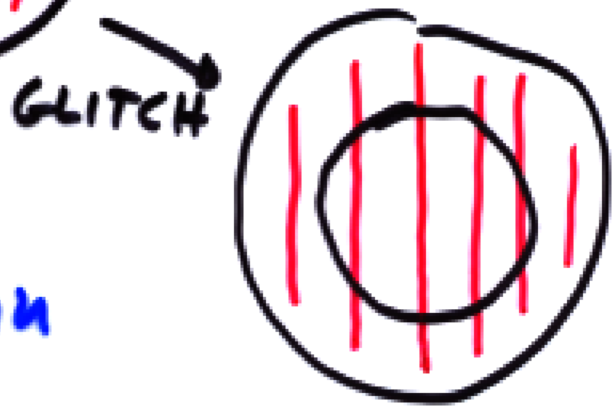


$$\frac{\delta\Omega}{\Omega} \sim 10^{-9} \to 10^{-6}$$

Conventional mechanism:







Glitches require non-uniformity (ie crystal) to impede (pin) motion of vortices.

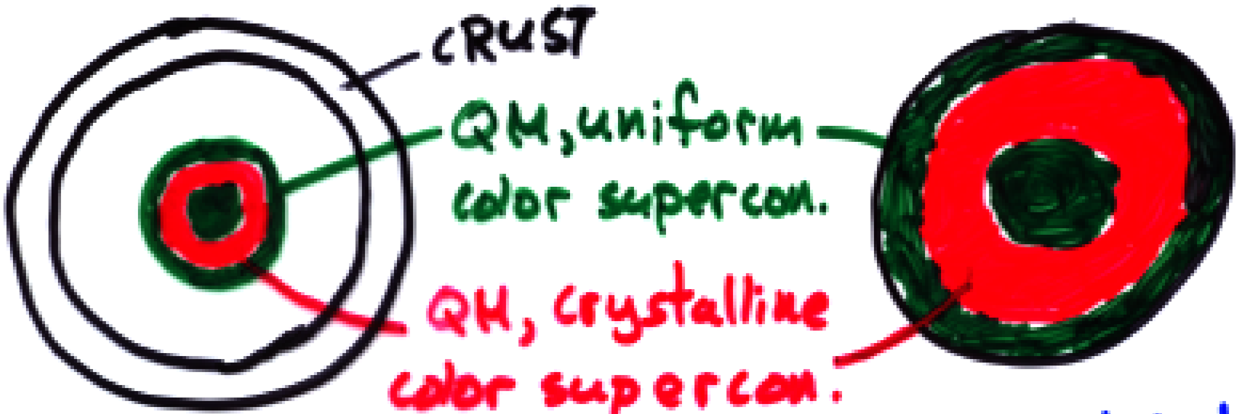
∴ thought impossible in QM.

GLITCHES IN QUARK MATTER?

- crystalline condensate will pin vortices.
- vortices prefer nodes of LOFF crystal
- crude estimates (dimensional analysis using F_{LOFF} & q) suggest that pinning force ~ comparable to that in nuclear crust. (Real calculation: future)

TWO IDEAS:

neutron star | strange quark star



larger glitches from crust; smaller glitches from QM core.

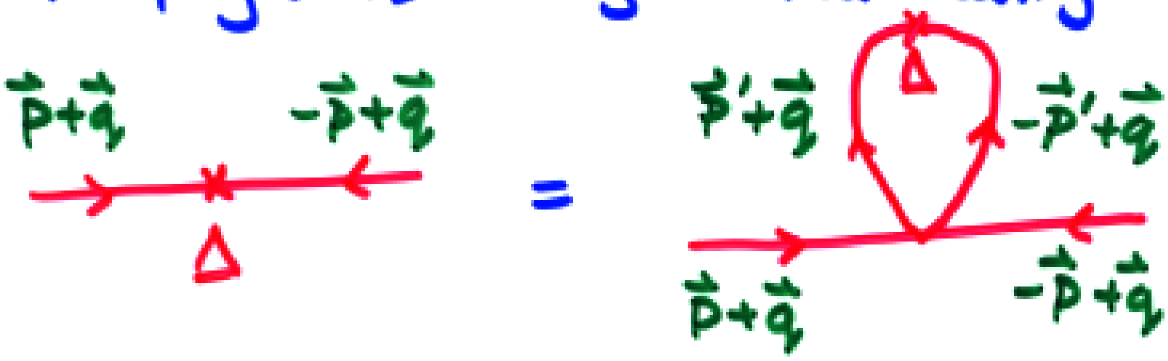
May glitch! Could pulsars be strange quark stars after all?

Real glitch phenomenology (eg are QM glitches same or different from crustal glitches) awaits better understanding of...

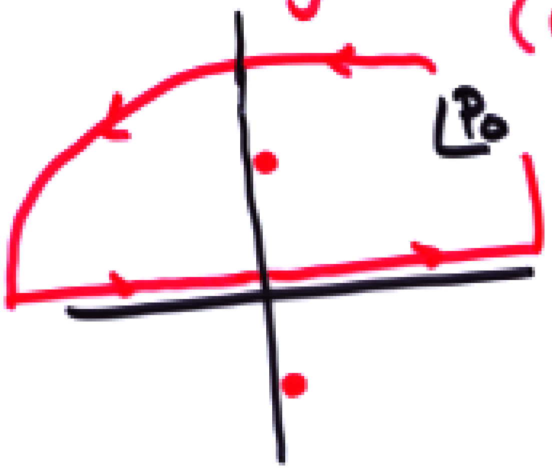
ONGOING AND FUTURE WORK

1.) Compare LOFF vs. $\langle uu\rangle$, $\langle dd\rangle$, $J=1$, uniform instead of LOFF vs. normal. Our results seem little changed. MA, JB, KR

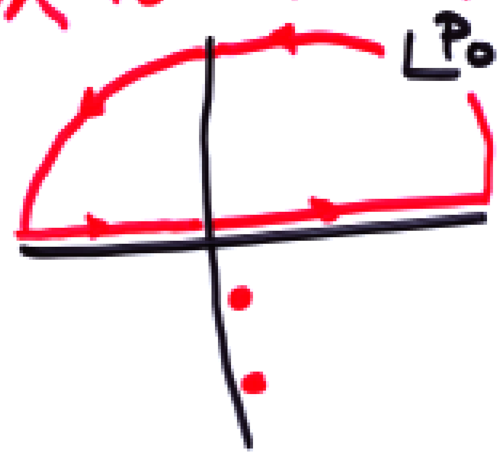
2.) Rederive gap equation via Gorkov propagators diagrammatically: done. JB, JK, KR, ES



$$\rightarrow \quad 1 = G\int d^4p \, \frac{\sin^2(\beta/2)}{(ip_0+E_1(\vec{p}))(ip_0-E_2(\vec{p}))}$$



vs.



$E_1>0$, $E_2>0 \rightarrow$ RHS $\neq 0$ $\qquad$ $E_1>0$, $E_2<0 \rightarrow$ RHS $=0$

Nice! Simplifies derivation of LOFF gap equation and allows us to generalize it:

3.) $T \neq 0$. $T_c(\delta\mu)$. Find $T_c \simeq 0.4\,\Delta$

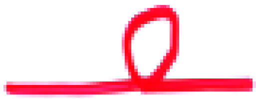
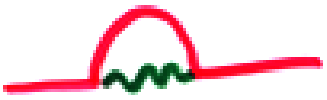
done. JB, JK, KR, ES

4.) What is optimal crystal structure??

in progress. JB, KR

5.) $M_s \neq 0$ and $\delta\mu \neq 0$.

in progress. JK, KR

6.)  →  in progress. KR, ES

Quarks interacting by gluon exchange

- Better estimate of LOFF window?
- $J=1$ attraction because...
- small angle scattering dominates
- quasi-1D physics at F.S.
- all suggest LOFF window widens
- controlled, weak-coupling, first principles QCD calculation of a crystalline phase!

7.) After 4, 5, 6: Can do real calculation of pinning force on rotational vortices → GLITCHES