

Lattice calculation of medium effects at short and long distances.

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Naive perturbation theory breaks down at $T \neq 0$.

Different length scales are generated at $T \neq 0$. For gauge theories $g \ll 1$ for $T \gg \Lambda_{\overline{MS}}$

$$\begin{array}{ccc} \frac{1}{T} & \ll & \frac{1}{gT} & \ll & \frac{1}{g^2T} \\ \uparrow & & \uparrow & & \uparrow \\ \text{Naive PT} & & \text{HTL resummed PT} & & \text{Non-perturbative methods} \end{array}$$

For interesting temperature range $g \gtrsim 1$.

For short distances $r \ll T$ no temperature dependence (medium effects) should be observed.

Open questions:

- Up to which length (momentum) scale perturbation theory is applicable ?
- At which scale medium effects (e.g. screening) becomes important ?

- **Static magnetic propagators in $SU(2)$ gauge theory**
A. Cucchieri, F. Karsch and P.P., *Phys. Lett.* **497** (2001) 80
- **Heavy quark potential in $SU(3)$ gauge theory**
F. Karsch, F. Zantow and P.P
- **Temporal quark and gluon propagators in quenched QCD**
F. Karsch, E. Laermann, S. Stikan, I. Wetzorke and P.P
- **Conclusion and outlook**

Magnetic propagators in SU(2) gauge theory at finite temperature

- Generation of a magnetic mass of order $g^2 T$ in non-Abelian gauge theories at $T \neq 0$ was postulated to avoid IR divergencies in PT. colorgreen (Linde)
- Self consistent resummation of PT :
for SU(2) $m_M = (0.28 - 0.38)g^2 T$
(Buchmüller and Philipsen, Alexanian and Nair, Eberlein)
- Lattice calculation of the Landau gauge propagators:
 $m_M = 0.46(3)g^2 T$ (Heller, Karsch and Rank, Karsch, Oevers, Petreczky)

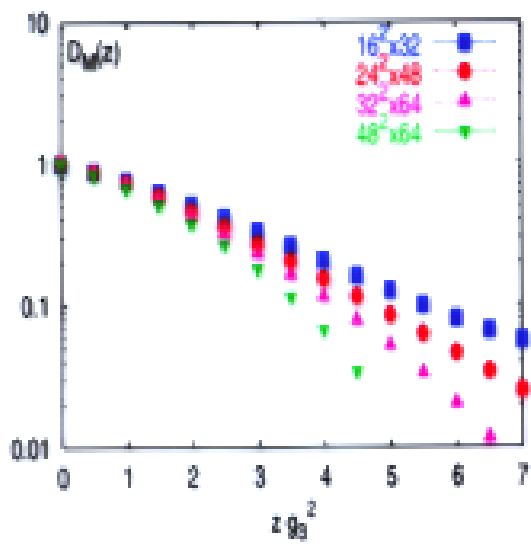
Magnetic propagators were studied in the dimensionally reduced 3d effective theory.

Dimensionfull coupling : $g_3^2 = g^2(T)T$.

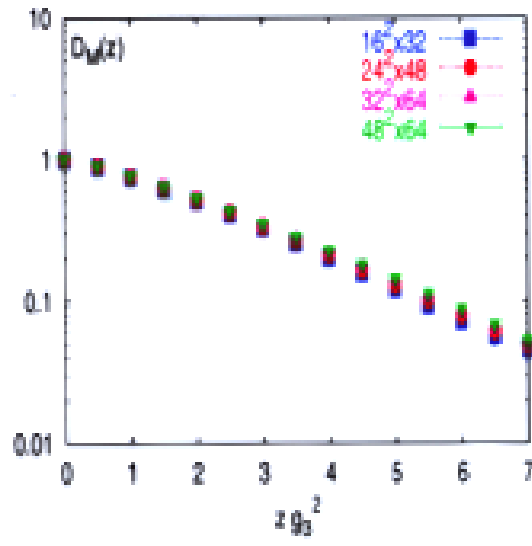
λ -gauges : $\partial_1 A_1 + \partial_2 A_2 + \lambda \partial_3 A_3 = 0$

$\lambda = 1$ is the Landau gauge

Maximally Abelian Gauge (MAG)



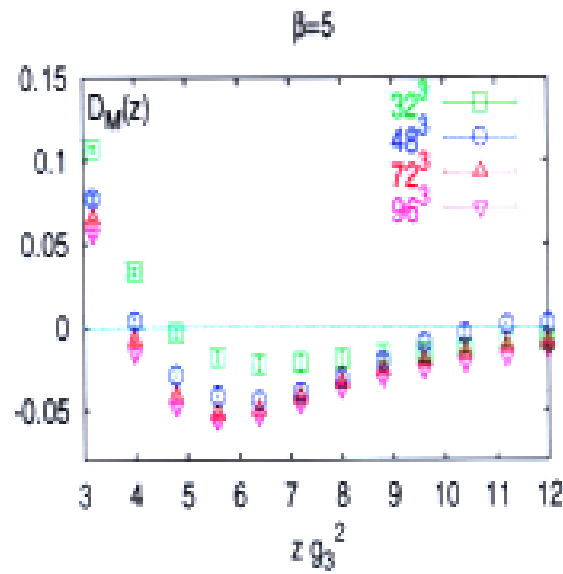
Landau



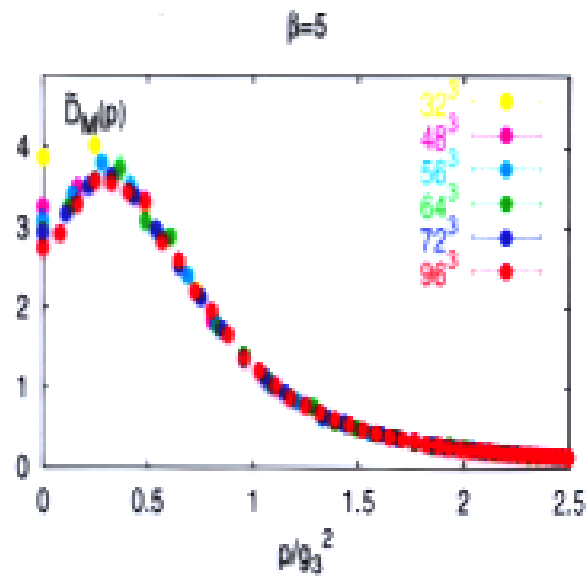
MAG

$$\beta = 4/g_3^2 a$$

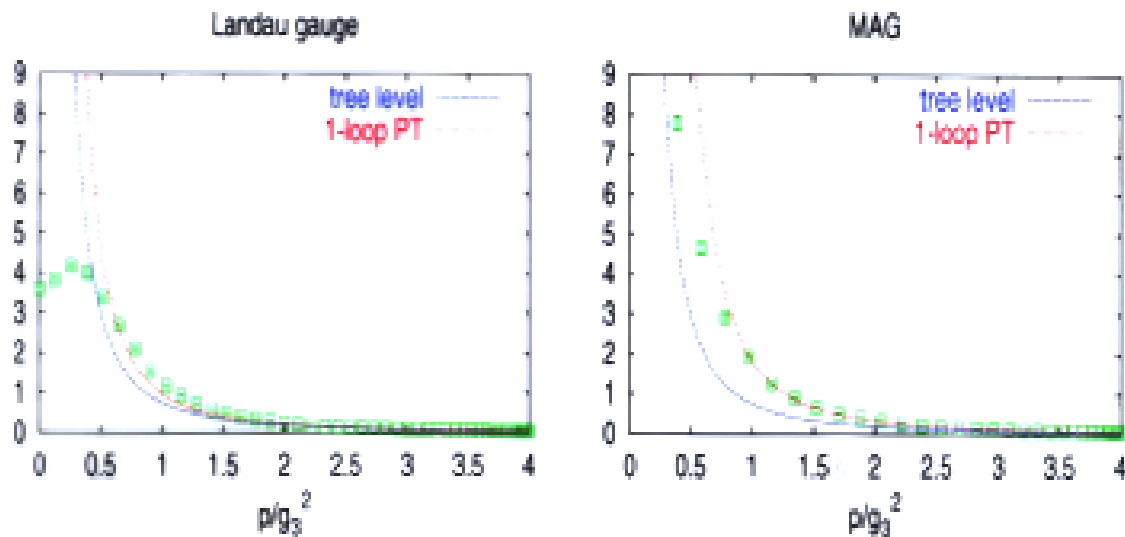
In λ -gauges the propagators become negative at large distances.



Momentum space magnetic propagators are infrared suppressed.



Comparison with perturbation theory



Perturbation theory works well for $p/g_3^2 > 1$ and breaks down completely at $p/g_3^2 \sim 0.7g_3^2$.

The heavy quark potential in SU(3) gauge theory

$$V(R) = -\log \frac{\langle L(R)L^\dagger(0) \rangle}{|\langle L \rangle|^2}$$

$L(R)$ is the Polyakov loop

- Most of studies concentrate on long distance behaviour of the heavy quark-potential $RT > 1$. For physics of heavy quarkonia, J/ψ , Υ etc. it is important to know the potential for $RT < 1$.
- At leading order in perturbation theory

$$\frac{V(R)}{T} = \frac{\alpha^2}{9} \frac{1}{(RT)^2}$$

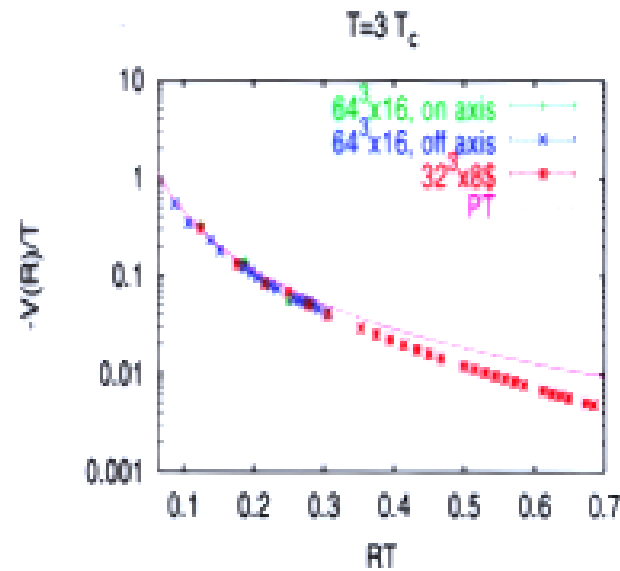
$\alpha = g^2/(4\pi)$ At large distances perturbation theory needs to be resummed leading to

$$\frac{V(R)}{T} = \frac{\alpha^2}{9} \frac{1}{(RT)^2} e^{-m_D R}, \quad RT \gg 1$$

$$m_D = gT \cdot 2$$

- Simulation with the Wilson action on $48^3 \times 12$ and $64^3 \times 16$ at $T = 1.5T_c$ and $3T_c$. Simulation with (2×1) improved action on $32^3 \times 8$ at $T/T_c = 1, 1.05, 1.14, 1.23, 1.50, 1.68, 3.0, 6.0, 12.0$

- Comparison of lattice data with PT.

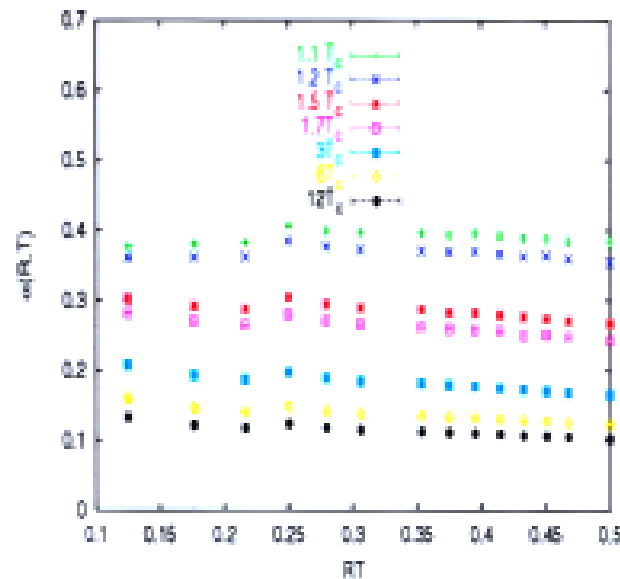


$$g(3T_c) = 1.6$$

$$g(1.5T_c) = 2.1$$

at $RT = 0.125$

- The running coupling constant Define $\alpha(R, T) = -9R^2TV(R, T)$



Temperature dependence of coupling:

$$(4\pi\alpha)^{-1} = \frac{11}{8\pi^2} \log(T/\Lambda_V) + \frac{51}{(88\pi^2)} \log(2 \log(T/\Lambda_V))$$

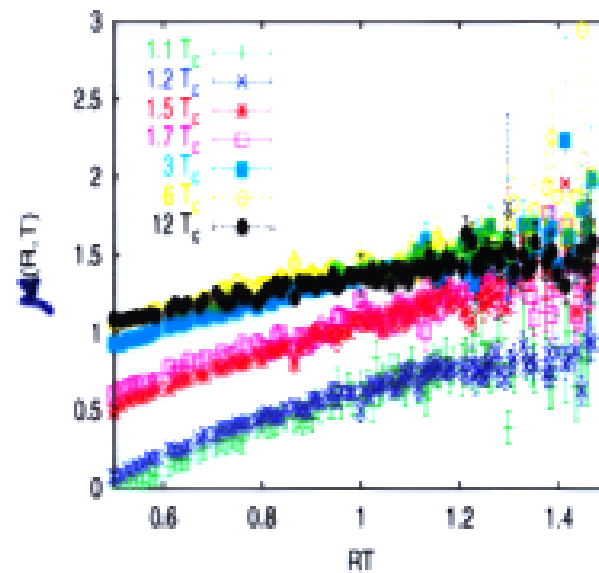
$$\Lambda_V \sim 0.4T_c$$

Expected: $\alpha(R, T)$ is only function of R for $RT \ll 1$

- Effective screening mass:

$$\mu(R) = -\frac{1}{R} \log\left(\frac{9R^2 TV(R, T)}{-\alpha^2}\right)$$

with α defined at $RT = 0.125$



The effective screening masses are smaller than the perturbative value $2gT$.

Reason: The momentum dependence of the gluon self-energy cannot be neglected for $RT < 1.5$.

J. C. Gale and J. Kapusta, Phys. Lett. B 198 (1987) 89

Temporal quark and gluon propagator at finite temperature in quenched QCD

- Temporal propagators carry information about the quasiparticle in the medium.
- Simulation were done on $64^3 \times 16$ lattice at $\beta = 7.457$ and $\kappa = 0.1339$ corresponding to $T = 3T_c$ and the quark mass $m_q \sim 30MeV$.
- Wilson action for the gauge sector, O(a)-improved Wilson fermions
- Calculating the propagator in the mixed (τ, p) representation one can extract the spectral function $\rho(\omega)$ with the help of the **Maximal Entropy Method (MEM)**

$$G(\tau, p) = \int_{-\infty}^{\infty} d\omega \rho(\omega, p) K(\tau, \omega)$$

$$K(\tau, \omega) = \frac{e^{-\omega\tau}}{1 \pm e^{-\omega\tau}}$$

- MEM works only for positive $\rho(\omega, p) \Rightarrow$ **Coulomb gauge**
- The position of peaks in the spectral function $\rho(\omega, p)$ gives the dispersion relation $\omega_p = \omega(p)$ of quasiparticles.
- The minimal spatial momentum available on our lattice is $p_{min} = 1.5T$.

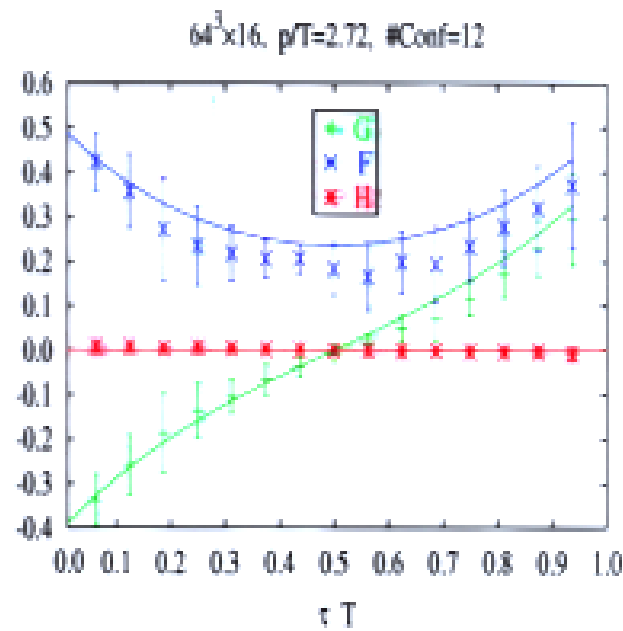
Quark propagators

$$S(\tau, p) = \gamma_0 F(\tau, p) + \vec{\gamma} \cdot \vec{p} G(\tau, p) + H(\tau, p)$$

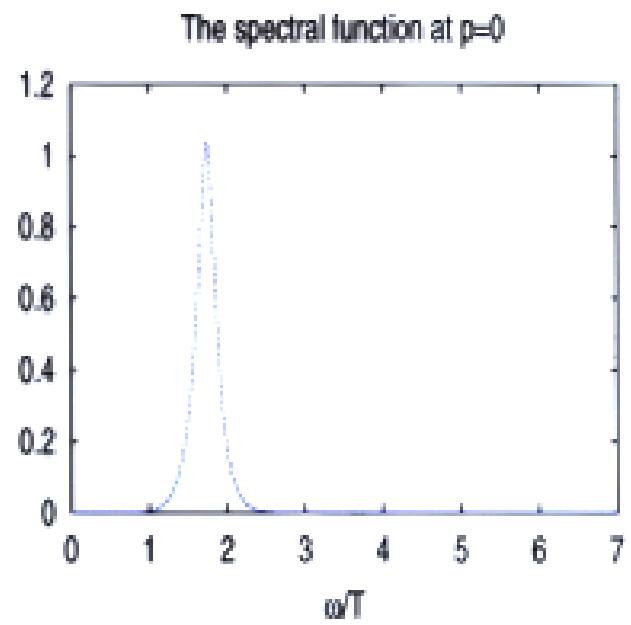
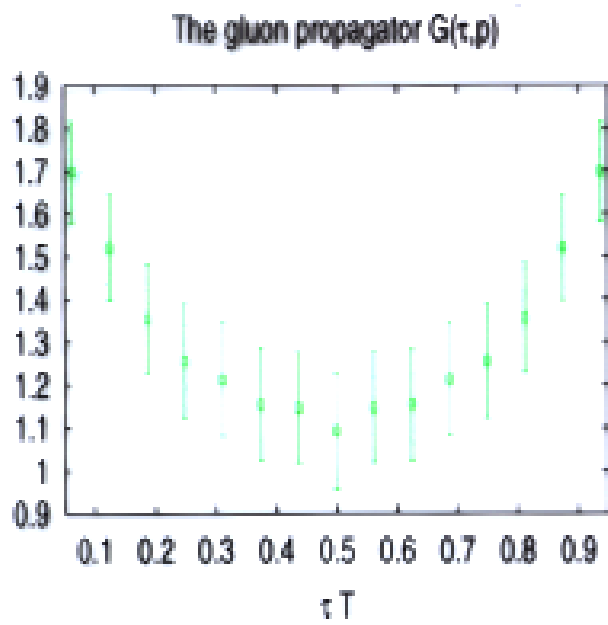
H is present due to breaking of the chiral symmetry on lattice.

Analysis was done for $p = 0$ and for $p = 2.72T$:

- At $p = 0$ only $\gamma_0 F(\tau, p)$ contributes. Results are compatible with HTL predictions.
- $p=2.72 T$



Gluon propagators at $p = 0$



$$\omega_p \sim 1.65T$$

HTL prediction : $\omega_p = 0.96$ using $g = 1.65$ from the heavy quark potential.

Conclusions and outlook

- Static magnetic propagators show quite different behaviour in different gauges. In Landau type gauges the data do not support the existence of a simple pole mass. PT breaks down at $p/g_3^2 \sim 0.7$ in all gauges considered.
 - The short distance behaviour of the heavy quark potential ($RT \lesssim 0.4$) is described by the leading order formula. Its temperature dependence can be viewed as temperature dependence of the coupling constant.
 - Screening effects become important at $RT > 0.5 \Rightarrow$
0.2 fm for $1.5 T_c$
0.1 fm for $3 T_c$
 - Temporal quark propagator are compatible with HTL results for zero and non-zero spatial momenta
 - Temporal gluon propagator at zero momenta shows considerable deviation from the HTL results.
- More statistics for temporal correlators is needed
- Complete 1-loop calculation of the heavy quark potential.