Lattice calculation of medium effects at short and long distances.

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Naive perturbation theory breaks down at \( T \neq 0 \).
Different length scales are generated at \( T \neq 0 \). For gauge theories \( g \ll 1 \) for \( T \gg \Lambda_{\overline{MS}} \)

\[
\frac{1}{T} \ll \frac{1}{gT} \ll \frac{1}{g^2T}
\]

\[
\uparrow \quad \uparrow \quad \uparrow
\]
Naive PT HTL Non-perturbative
resummed methods
PT

For interesting temperature range \( g \gtrsim 1 \).
For short distances \( r \ll T \) no temperature dependence (medium effects) should be observed.

Open questions:

- Up to which length (momentum) scale perturbation theory is applicable?
- At which scale medium effects (e.g. screening) becomes important?
- Static magnetic propagators in SU(2) gauge theory

- Heavy quark potential in SU(3) gauge theory
  F. Karsch, F. Zantow and P.P.

- Temporal quark and gluon propagators in quenched QCD
  F. Karsch, E. Laermann, S. Stikan, I. Wetzorke and P.P.

- Conclusion and outlook
Magnetic propagators in SU(2) gauge theory at finite temperature

- Generation of a magnetic mass of order $g^2T$ in non-Abelian gauge theories at $T \neq 0$ was postulated to avoid IR divergencies in PT. (Linde)

- Self consistent resummation of PT:
  for SU(2) $m_M = (0.28 - 0.38)g^2T$
  (Buchmüller and Philipsen, Alexanian and Nair, Eberlein)

- Lattice calculation of the Landau gauge propagators:
  $m_M = 0.46(3)g^2T$ (Heller, Karsch and Rank, Karsch, Oevers, Petreczky)

Magnetic propagators were studied in the dimensionally reduced 3d effective theory.
Dimensionfull coupling : $g_3^2 = g^2(T)T$.

$\lambda$ -gauges : $\partial_1 A_1 + \partial_2 A_2 + \lambda \partial_3 A_3 = 0$
$\lambda = 1$ is the Landau gauge
Maximally Abelian Gauge (MAG)
\( \beta = \frac{4}{g_3^2 a} \)

In \( \lambda \)-gauges the propagators become negative at large distances.
Momentum space magnetic propagators are infrared suppressed.

\[ \beta = 5 \]

\[ \tilde{\delta}_M(p) \]

\[ p/g_3^2 \]

Comparison with perturbation theory

<table>
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<th>Landau gauge</th>
<th>MAG</th>
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Perturbation theory works well for \( p/g_3^2 > 1 \) and breaks down completely at \( p/g_3^2 \sim 0.7g_3^2 \).
The heavy quark potential in SU(3) gauge theory

\[ V(R) = -\log \frac{\langle L(R)L^\dagger(0) \rangle}{|\langle L \rangle|^2} \]

\( L(R) \) is the Polyakov loop

- Most of studies concentrate on long distance behaviour of the heavy quark-potential \( RT > 1 \). For physics of heavy quarkonia, \( J/\psi, \Upsilon \) etc. it is important to know the potential for \( RT < 1 \).

- At leading order in perturbation theory

\[ \frac{V(R)}{T} = \frac{\alpha^2}{9} \frac{1}{(RT)^2} \]

\( \alpha = g^2/(4\pi) \) At large distances perturbation theory needs to be resummed leading to

\[ \frac{V(R)}{T} = \frac{\alpha^2}{9} \frac{1}{(RT)^2} e^{-m_D R}, \quad RT \gg 1 \]

\( m_D = gT \cdot 2 \)

- Simulation with the Wilson action on \( 48^3 \times 12 \) and \( 64^3 \times 16 \) at \( T = 1.5T_c \) and \( 3T_c \). Simulation with \((2 \times 1)\) improved action on \( 32^3 \times 8 \) at \( T/T_c = 1, 1.05, 1.14, 1.23, 1.50, 1.68, 3.0, 6.0, 12.0 \)
- Comparison of lattice data with PT.

\[ g(3T_c) = 1.6 \]
\[ g(1.5T_c) = 2.1 \]
\[ \text{at } RT = 0.125 \]

- The running coupling constant Define \( \alpha(R, T) = -9R^2TV(R, T) \)

Temperature dependence of coupling:

\[
(4\pi\alpha)^{-1} = \frac{11}{8\pi^2} \log\left(\frac{T}{\Lambda_V}\right) + \frac{51}{(88\pi^2)} \log(2 \log(\frac{T}{\Lambda_V}))
\]

\( \Lambda_V \sim 0.4T_c \)

Expected: \( \alpha(R, T) \) is only function of \( R \) for \( RT \ll 1 \)

- Effective screening mass:
\[ \mu(R) = -\frac{1}{R} \log\left( \frac{9R^2 TV(R, T)}{-\alpha^2} \right) \]

with \( \alpha \) defined at \( RT = 0.125 \)

The effective screening masses are smaller than the perturbative value \( 2gT \).

Reason: The momentum dependence of the gluon self-energy cannot be neglected for \( RT < 1.5 \).

Temporal quark and gluon propagator at finite temperature in quenched QCD

- Temporal propagators carry information about the quasiparticle in the medium.
- Simulation were done on $64^3 \times 16$ lattice at $\beta = 7.457$ and $\kappa = 0.1339$ corresponding to $T = 3T_c$ and the quark mass $m_q \sim 30MeV$.
- Wilson action for the gauge sector, $O(a)$-improved Wilson fermions
- Calculating the propagator in the mixed $(\tau, p)$ representation one can extract the spectral function $\rho(\omega)$ with the help of the Maximal Entropy Method (MEM)

$$G(\tau, p) = \int_{-\infty}^{\infty} d\omega \rho(\omega, p) K(\tau, \omega)$$

$$K(\tau, \omega) = \frac{e^{-\omega \tau}}{1 \pm e^{-\omega \tau}}$$

- MEM works only for positive $\rho(\omega, p) \Rightarrow$ Coulomb gauge
- The position of peaks in the spectral function $\rho(\omega, p)$ gives the dispersion relation $\omega_p = \omega(p)$ of quasiparticles.
- The minimal spatial momentenmum available on our lattice is $p_{\text{min}} = 1.5T$. 
Quark propagators

\[ S(\tau, p) = \gamma_0 F(\tau, p) + \vec{\gamma} \cdot \vec{p} G(\tau, p) + H(\tau, p) \]

\( H \) is present due to breaking of the chiral symmetry on lattice.

Analysis was done for \( p = 0 \) and for \( p = 2.72T \):

- At \( p = 0 \) only \( \gamma_0 F(\tau, p) \) contributes. Results are compatible with HTL predictions.
- \( p = 2.72 \text{ T} \)
Gluon propagators at $p = 0$

The gluon propagator $G(\tau, p)$

The spectral function at $p=0$

$\downarrow$

$\omega_p \sim 1.65T$

HTL prediction: $\omega_p = 0.96$ using $g = 1.65$ from the heavy quark protential.
Conclusions and outlook

- Static magnetic propagators show quite different behaviour in different gauges. In Landau type gauges the data do not support the existence of a simple pole mass. PT breaks down at $p/g_3^2 \approx 0.7$ in all gauges considered.

- The short distance behaviour of the heavy quark potential ($RT \lesssim 0.4$) is described by the leading order formula. Its temperature dependence can be viewed as temperature dependence of the coupling constant.

- Screening effects become important at $RT > 0.5 \Rightarrow 0.2 \text{ fm for } 1.5T_c \Rightarrow 0.1 \text{ fm for } 3T_c$

- Temporal quark propagator are compatible with HTL results for zero and non-zero spatial momenta.

- Temporal gluon propagator at zero momenta shows considerable deviation from the HTL results.

- More statistics for temporal correlators is needed.

- Complete 1-loop calculation of the heavy quark potential.