The Thermodynamics of the Quark-Gluon Plasma: Self-consistent Resummations vs. Lattice Data

- "First principle" calculation of the Entropy (or the Quark Density) of the Quark-Gluon Plasma using analytical techniques.
  Good agreement with the lattice results down to temperatures $T \approx 2.5 T_c$.
- Work done in collaboration with Jean-Paul Blaizot (SPhT Saclay) and Tony Rebhan (TU Wien).

Phys. Rev. Lett. **83**, 2906 (1999)

Phys. Lett. **B 470**, 181 (1999)

hep-ph/0005003 (to appear in Phys. Rev. **D**)

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INTRODUCTION

• Lattice QCD : The most compelling theoretical evidences for the existence of the Quark-Gluon Plasma

$T_c \approx 270$ Mev for pure Yang-Mills theory (no quarks)

$T_c \approx 170$ Mev (2 or 3 flavors of light quarks)

[cf. talk by F. Karsch]

• High Temperature ($T \gg T_c$): Asymptotic freedom

$\Longrightarrow$ Weakly interacting gas of "quasiparticles" (quarks and gluons)

$$\alpha_s(\bar{\mu}) \equiv \frac{g^2}{4\pi} \propto \frac{1}{\ln(\bar{\mu}/\Lambda_{QCD})}, \quad \bar{\mu} = 2\pi T$$

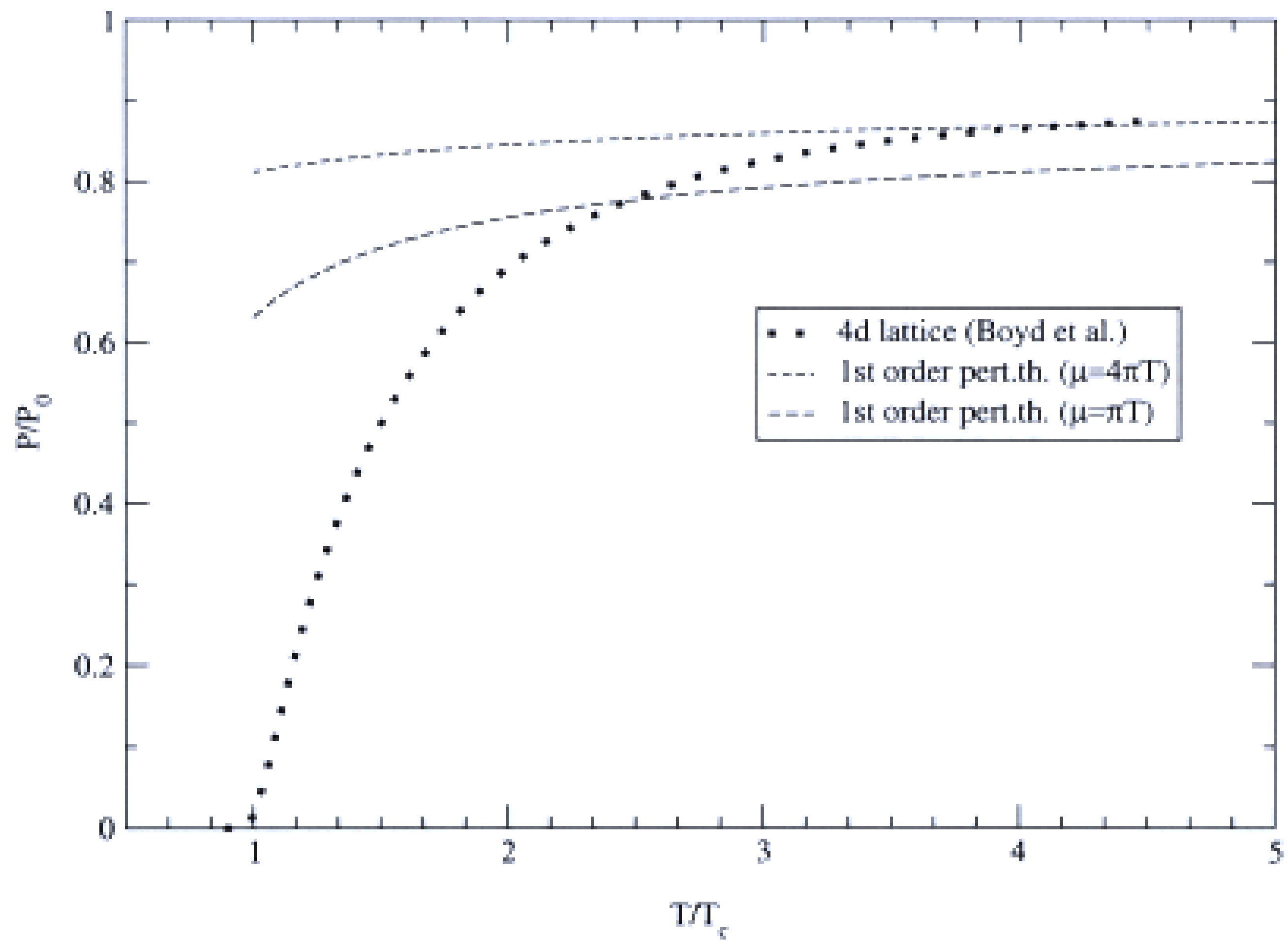
$$\begin{cases} \alpha_s \approx 0.3 & \text{for} \quad T \simeq 0.3\,\mathrm{GeV} \simeq T_c \\ \alpha_s \approx 0.05 & \text{for} \quad T \simeq 10^5\,\mathrm{GeV} \end{cases}$$

• Supported by lattice calculations:

$$\frac{P - P_0}{P_0} \lesssim 25\% \quad \text{for} \quad T \gtrsim 2T_c$$

$\Longrightarrow$ Weak coupling methods

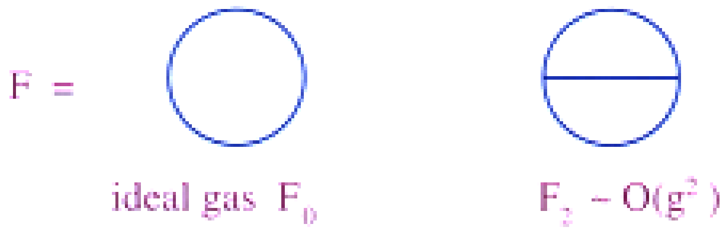
Perturbation theory looks promising at order g^2 ...



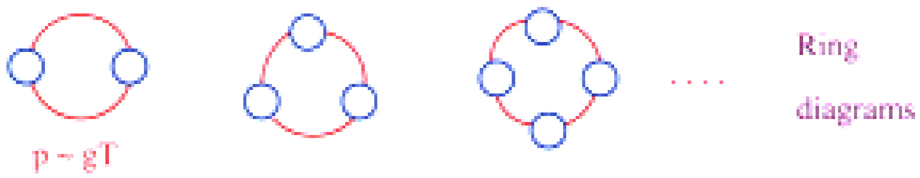
4d lattice (Boyd et al.)
1st order pert.th. (μ=4πT)
1st order pert.th. (μ=πT)
P/P0
T/Tc
0
0.2
0.4
0.6
0.8
1
2
3
4
5

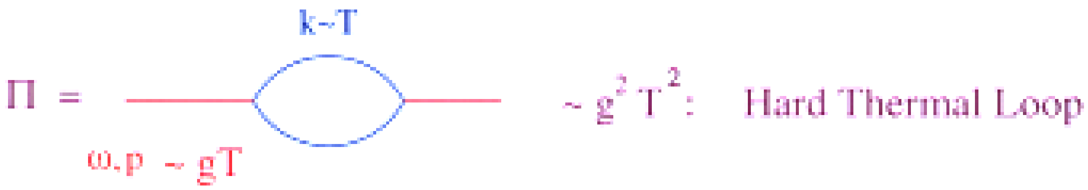
The Quasiparticle Picture

- The plasma particles have $k \sim T$ ("hard") and give the dominant contribution to the thermodynamics (e.g., to $\mathcal{F}$).

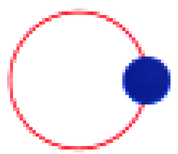


- The "soft" fields ($k \sim gT$) are collective excitations and start contributing to $\mathcal{F}$ at order g^3.





$$D_0 = \frac{1}{\omega^2 - p^2} \longrightarrow D = \frac{1}{\omega^2 - p^2 - \Pi(\omega, p)}, \quad p^2 \sim \Pi$$



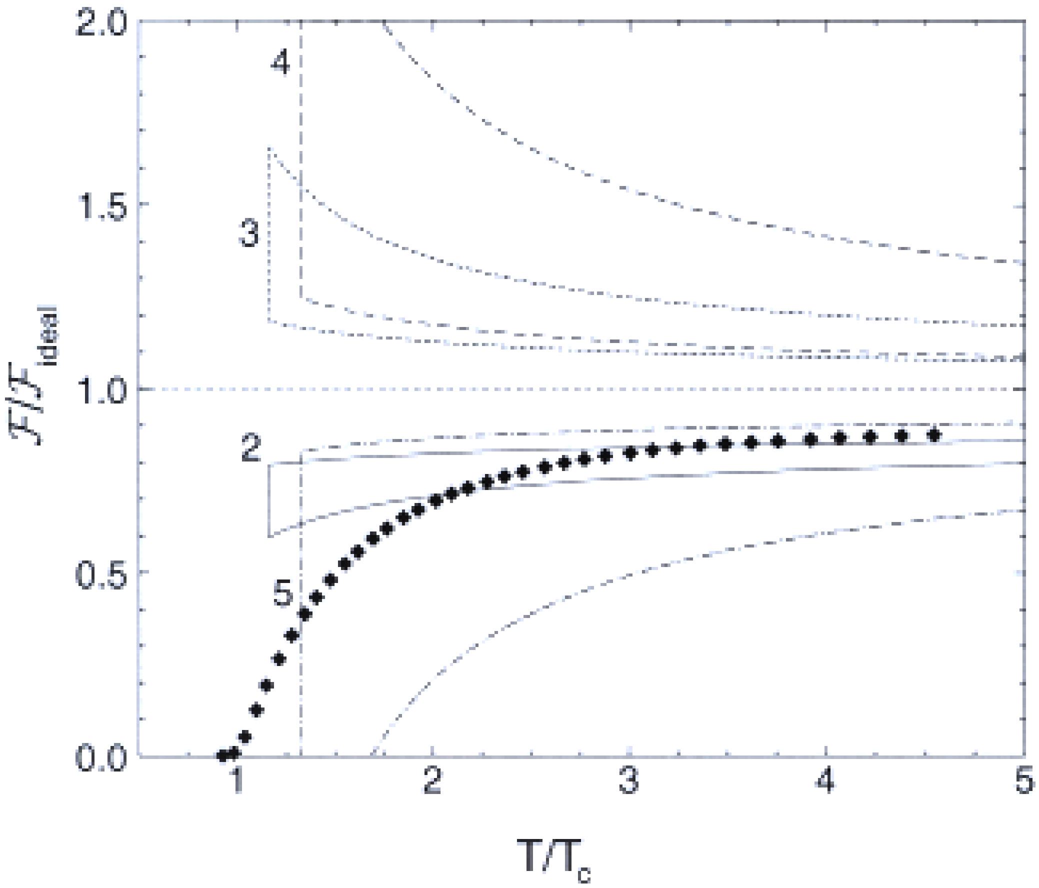
$$\sim O(g^3) + g^4 \ln \frac{1}{g} + g^4 + g^5 + \ldots$$

- $\mathcal{F}_{soft}$ is a non-analytic function of Π (or g^2)

... but fails badly when higher orders are included!

- The weak coupling expansion of the free energy $(\mathcal{F})$ is known up to order $\alpha_s^{5/2}$ (or g^5).

[Arnold & Zhai (94,95), Zhai & Kastening (95)]



- Strict perturbative expansion is manifestly useless !
- Odd powers of g : Infinite resummations of the soft fields: $k \sim gT \ll T$.
- The most dramatic fluctuations come from the soft modes. [Braaten and Nieto (96)]

Beyond Perturbation Theory

It makes no sense to treat the soft modes in (strict) perturbation theory:

- Expansion in powers of g, but ... $g \simeq 1 \cdots 2$ for $T = 2T_c \cdots 10T_c$!
- The collective modes are non-perturbative (they do not even exist in the absence of interactions)

"Weak coupling methods are useless except at asymptotically high temperatures (where $g \ll 1$)."
WRONG !

• Physicswise, one expects the overall contribution of the soft modes to the thermodynamics to be small

— Recent numerical evidence [dimens. reduction + 3dim lattice]
[Kajantie et al., PRL 86 (2001) 10; cf. talk by Y. Schroder]

• Resummations which attempt to capture the correct analytic dependence on g at large T ($g \ll 1$) and extrapolate it towards lower T ($g \sim 1$).

— Andersen, Braaten, and Strickland [PRL 83 (1999) 2139]
[based on "screened perturbation theory" by Karsch et al., 97]

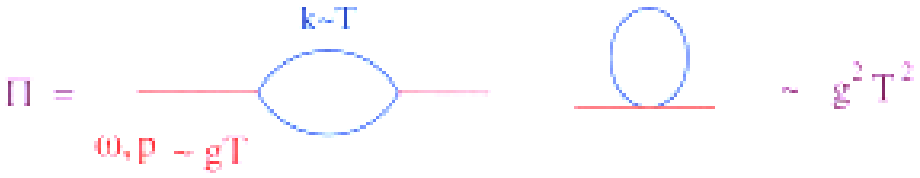
— Blaizot, Rebhan, and E.I. [PRL 83 (1999) 2906]

Express the thermodynamical quantities in terms of the HTL–resummed propagators (and vertices)

Hard Thermal Loops

[Braaten and Pisarski (90), Frenkel and Taylor (90)]

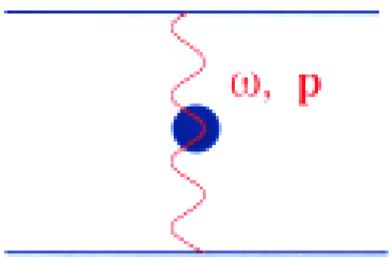
- Large ($\propto g^2T^2$) thermal "corrections" to the amplitudes of the soft fields:



- Debye screening:

$$\Pi_{el}(\omega \ll p) \simeq m_D^2 \Longrightarrow D_{el}(\omega \ll p) \simeq \frac{1}{p^2 + m_D^2}$$

- Dynamical screening : $D_{mag}(\omega \ll p) \simeq \dfrac{1}{p^2 - i\frac{\omega}{p}\, m_D^2}$



— transport coefficients [Baym et al.]

— quasiparticle damping rates [Blaizot et E.I.]

— baryon number violation rate at high T [Arnold, Son et Yaffe]

— colour superconductivity [Son]

- Quasiparticle poles $\Longrightarrow$ collective plasma modes

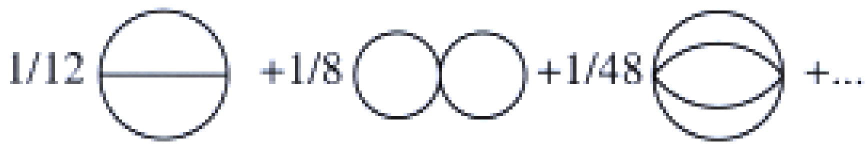
$$\omega^2 - p^2 - \Pi(\omega, p) = 0 \Longrightarrow \omega = \omega(p)$$

The Free Energy of Dressed Quasiparticles

- The "skeleton" representation of the free energy in terms of the full propagator D [Luttinger and Ward (60)] :

$$\mathcal{F}[D] = \frac{1}{2}\mathrm{Tr}\ln D^{-1} - \frac{1}{2}\mathrm{Tr}\Pi D + \Phi[D],$$

$\Phi[D] \equiv$ the sum of 2-particle irreducible diagrams.

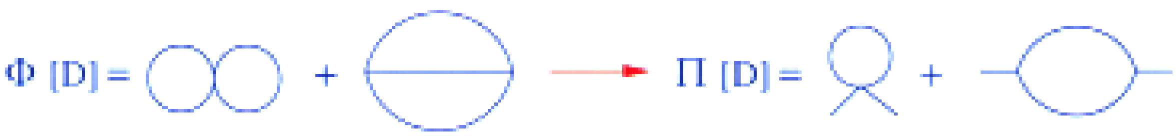
$\Phi[D] =$ 

$$\Pi[D] = 2\frac{\delta\Phi}{\delta D} \Longrightarrow \frac{\delta\mathcal{F}[D]}{\delta D} = 0.$$

- Self-consistent approximations [Baym (62)]

Select some particular skeletons in Φ and solve the Dyson equation self-consistently: $D^{-1} = D_0^{-1} + \Pi[D]$

- 2-loop approximation for $\Phi[D]$:



- The entropy $\mathcal{S}[D]$:

$$\mathcal{S} = -\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}T} = -\left.\frac{\partial\mathcal{F}}{\partial T}\right|_D$$

The 2-loop Entropy

$$\mathcal{S} = -\int \frac{d^4p}{(2\pi)^4} \frac{\partial N}{\partial T} \left\{ \mathrm{Im} \ln D^{-1} - \mathrm{Im}\Pi \,\mathrm{Re} D \right\}$$

where $N(\omega) \equiv \frac{1}{e^{\beta\omega}-1}$.

$\Longrightarrow$ Effectively one-loop expression.

$\Longrightarrow$ Manifestly ultraviolet-finite.

$\Longrightarrow$ Non perturbative (all orders in g).

$\Longrightarrow$ Perturbatively correct up to order g^3.

- Why entropy ?

$\mathcal{S}$ is most directly related to the quasiparticle spectrum:

— The residual interactions start contributing to $\mathcal{S}$ to order 3-loop, or g^4. [In $\mathcal{F}$: 2-loop, or $O(g^2)$]

— Up to order g^3, the self-energy of the hard particles is needed only on the light-cone: $\mathrm{Re}\,\Pi_{hard}(\omega^2 = p^2)$

$\rightarrow$ "thermal masses" for the plasma particles

$$\omega = p \longrightarrow \omega = \sqrt{p^2 + m_\infty^2}, \quad m_\infty \sim gT$$

$\Longrightarrow$ The best quasiparticle approximation one can get for the thermodynamics

- Reconstructing the pressure $\mathcal{P} = -\mathcal{F}$:

$$\mathcal{P}(T) = \int_{T_0}^{T} dT' \,\mathcal{S}(T') + \mathcal{P}(T_0)$$

with $\mathcal{P}(T_0)$ taken from the lattice data.

Approximately Self-Consistent Entropy

By itself, the self-consistent truncation is not a gauge invariant approximation.

- Replace self-consistency by gauge-invariant approximations to Π, correct up to order g^3.
- Compute the entropy exactly (i.e., numerically) with these approximations for Π.

— $\omega, p \sim gT$: $\Pi_{soft} \approx \Pi_{HTL}$

— $\omega, p \sim T$: $\Pi_{hard}(\omega^2 = p^2)$ to $\mathcal{O}(g^2)$ and $\mathcal{O}(g^3)$

All these approximations are gauge-invariant !

$$\Pi^{(2)}_{hard}|_{\omega=p} = \Pi_{HTL}|_{\omega=p} = m^2_{\infty} \sim (gT)^2$$

$$\Longrightarrow \quad \mathcal{S}_2 = -T\left\{\sum_B \frac{m^2_{B\,\infty}}{12} + \sum_F \frac{m^2_{F\,\infty}}{24}\right\}$$

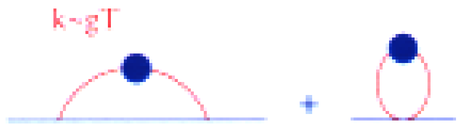
- The HTL, or leading, approximation:

$\Pi = \Pi_{HTL}$ at all momenta $\Longrightarrow$ $\boxed{\mathcal{S} = \mathcal{S}_{HTL}}$

Perturbative content : $\mathcal{O}(g^2) + \frac{1}{4}\,\mathcal{O}(g^3)$

- The next-to-leading approximation:

$\Pi_{soft} = \Pi_{HTL}$, $\Pi_{hard} = \Pi_{HTL} + \delta\Pi$ $\Longrightarrow$ $\boxed{\mathcal{S} = \mathcal{S}_{NLA}}$

$\delta\Pi(\omega = p) \equiv$ 

$= \mathrm{O}(g^3T^2)$

Perturbative content : $\mathcal{O}(g^2) + \mathcal{O}(g^3)$

RESULTS: The entropy for SU(3) Yang-Mills

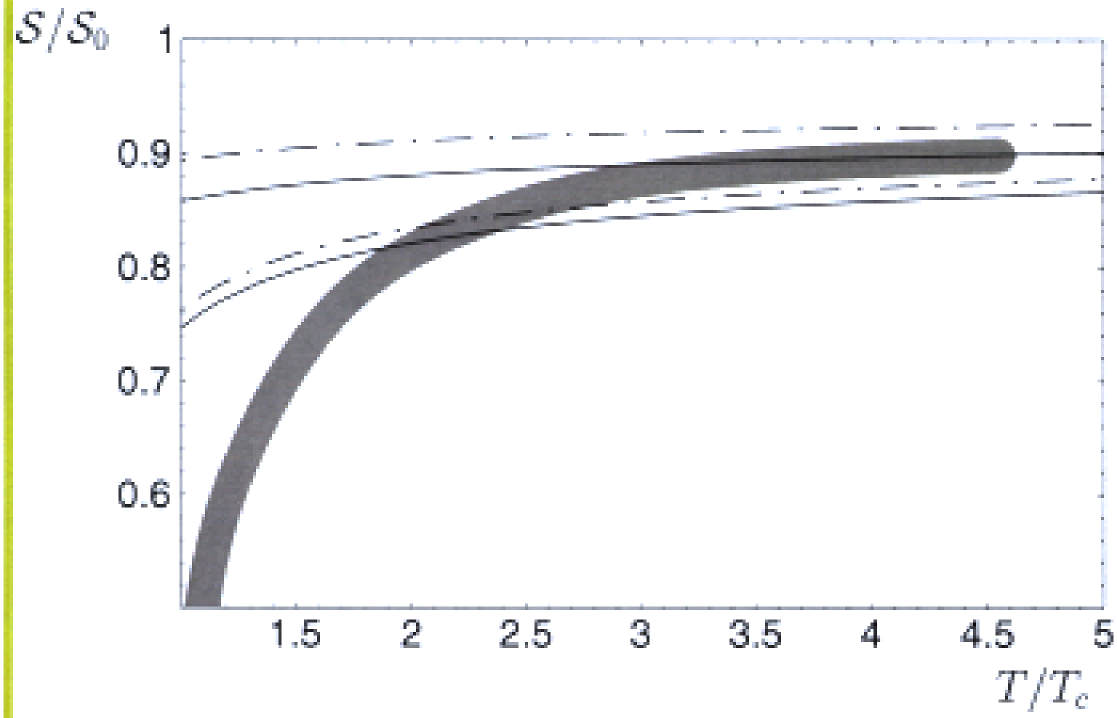


Figure 1: The entropy of pure SU(3) gauge theory normalized to the ideal gas entropy $\mathcal{S}_0$.

Full lines: $\mathcal{S}_{HTL}$. Dashed-dotted lines: $\mathcal{S}_{NLA}$.

2-loop β-function $\rightarrow$ the running coupling constant $\alpha_s(\bar{\mu})$.

The $\overline{\text{MS}}$ renormalisation scale: $\bar{\mu} = \pi T \cdots 4\pi T$.

The dark grey band: lattice result by Boyd et al (1996).

The Pressure for SU(3) Yang-Mills

$$\mathcal{P}(T) = \int_{T_c}^{T} \mathrm{d}T' \, \mathcal{S}(T') \; + \; \mathcal{P}(T_c), \quad \text{with } \mathcal{P}(T_c) \approx 0$$

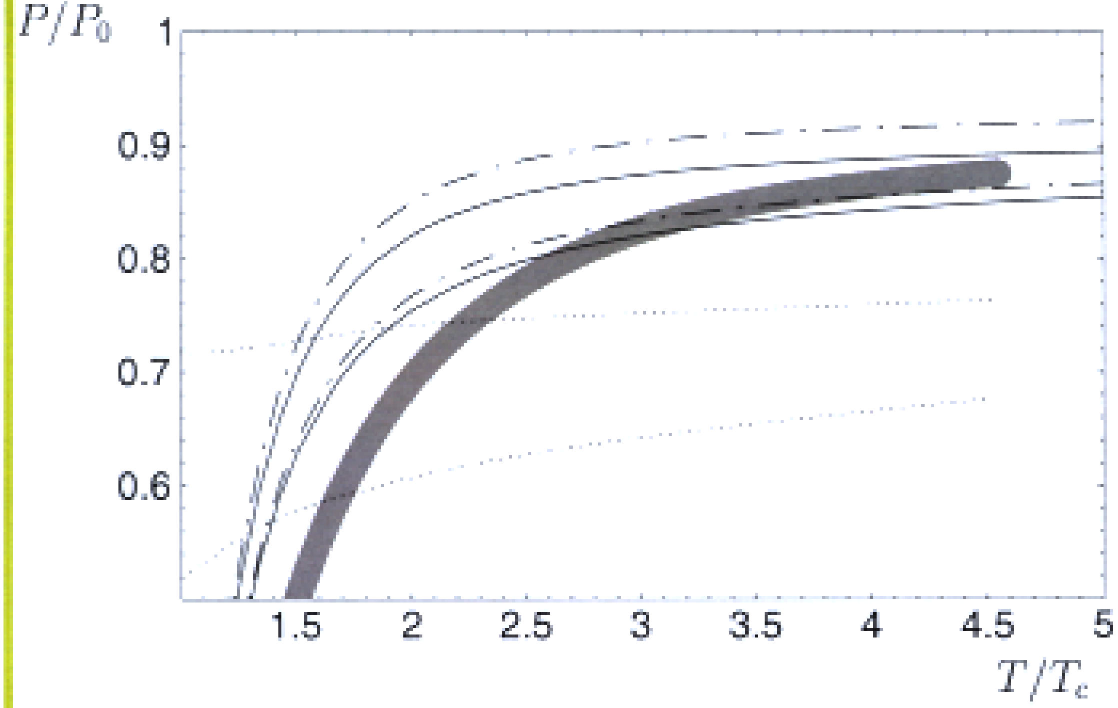


Figure 2: $\mathcal{P}/\mathcal{P}_0$ for pure SU(3) gauge theory.

Full lines: $\mathcal{P}_{HTL}$. Dashed-dotted lines: $\mathcal{P}_{NLA}$.

Dotted lines: results by Andersen et al. (1999).

The $\overline{\text{MS}}$ renormalisation scale: $\bar{\mu} = \pi T \cdots 4\pi T$.

The dark grey band: lattice result by Boyd et al (1996).

Quark-Gluon Plasma with two massless flavours

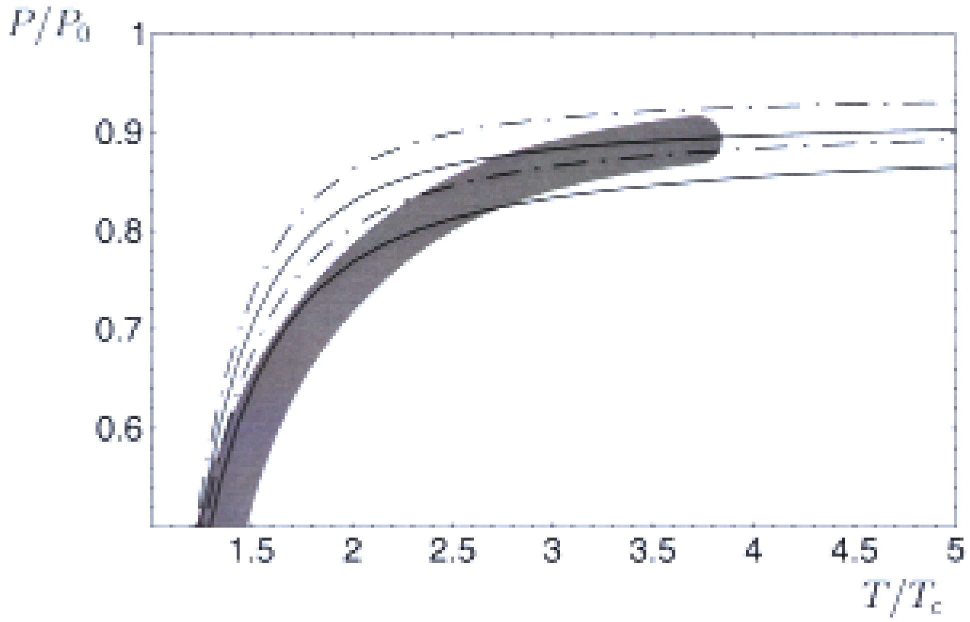


Figure 3: The pressure for a quark-gluon plasma with $N_f = 2$ versus the lattice results by Karsch et al. (2000). Full lines: $\mathcal{P}_{HTL}$. Dashed-dotted lines: $\mathcal{P}_{NLA}$.

The entropy for various N_f's: blown-up scale

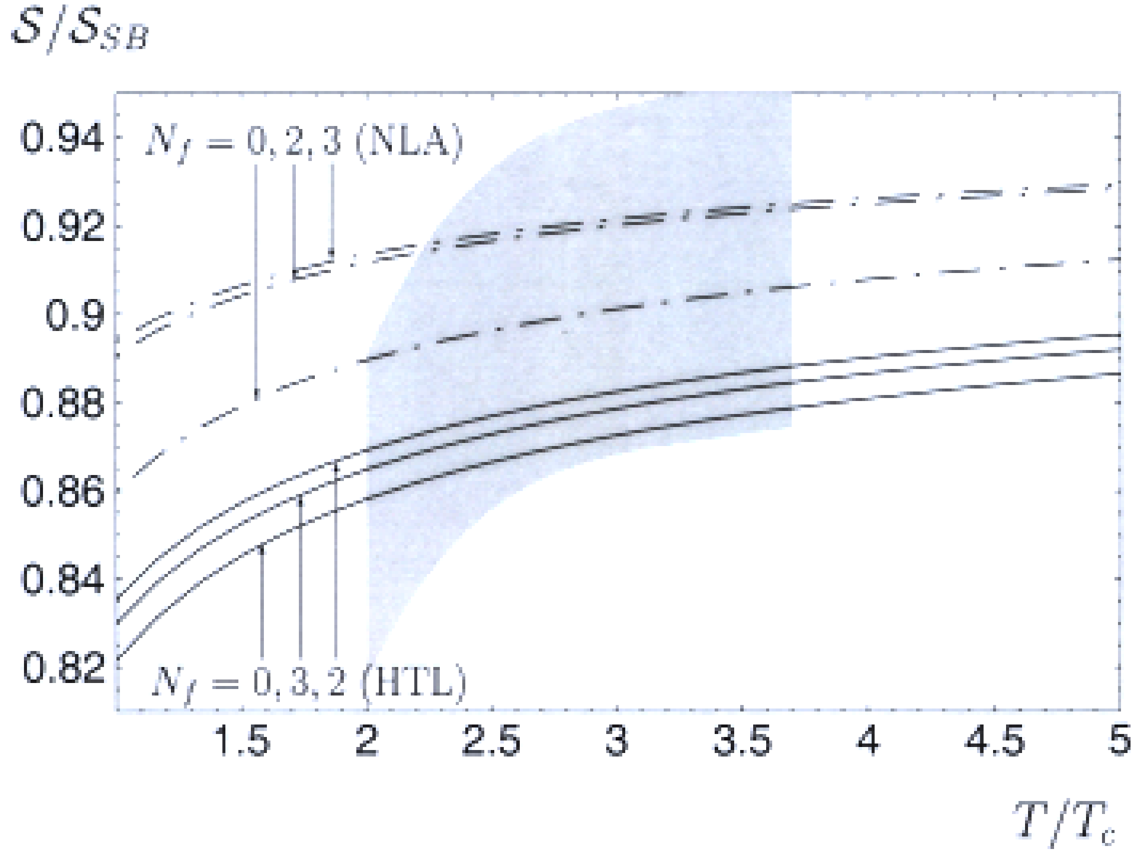
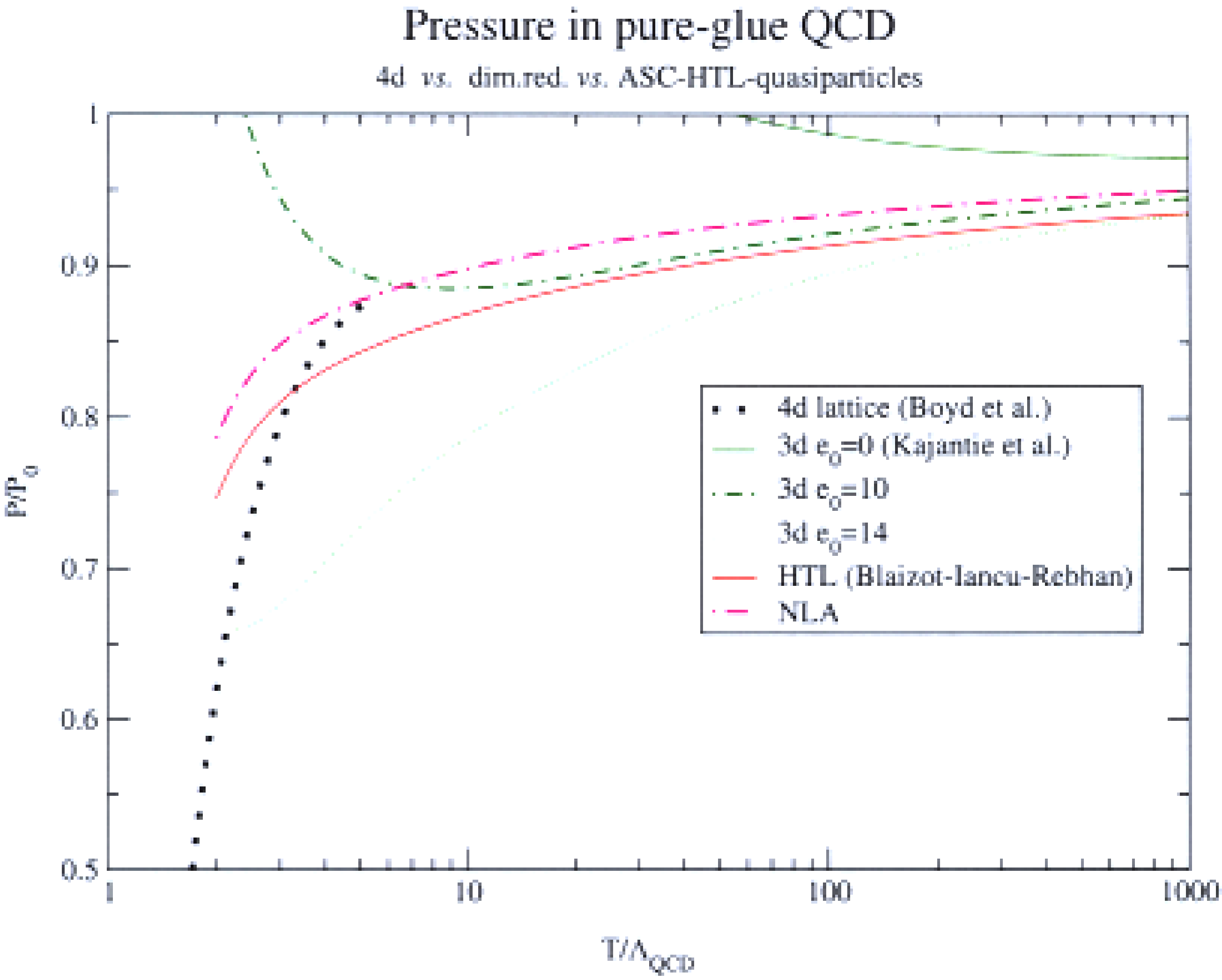


Figure 4: Comparison of $\mathcal{S}_{HTL}$ (full lines) and $\mathcal{S}_{NLA}$ (dash-dotted lines) for flavor numbers $N_f = 0, 2, 3$, all with $\bar{\mu} = 2\pi T$.

Gray band: continuum extrapolation of the lattice result for $N_f = 2$ by Karsch et al, 2000.

High Temperature Behaviour



Approximately Self-Consistent resummation

[Blaizot, Rebhan, and E.I (1999, 2001)]

vs. Dimensional Reduction

[Kajantie, Laine, Rummukainen, and Schroder (2000)]

Quark Density: $N_f = 3$

Cold Dense Quark Matter: $T = 0$, $\mu > 0$, $\mathcal{N} = -\frac{\partial \mathcal{F}}{\partial \mu}|_D$

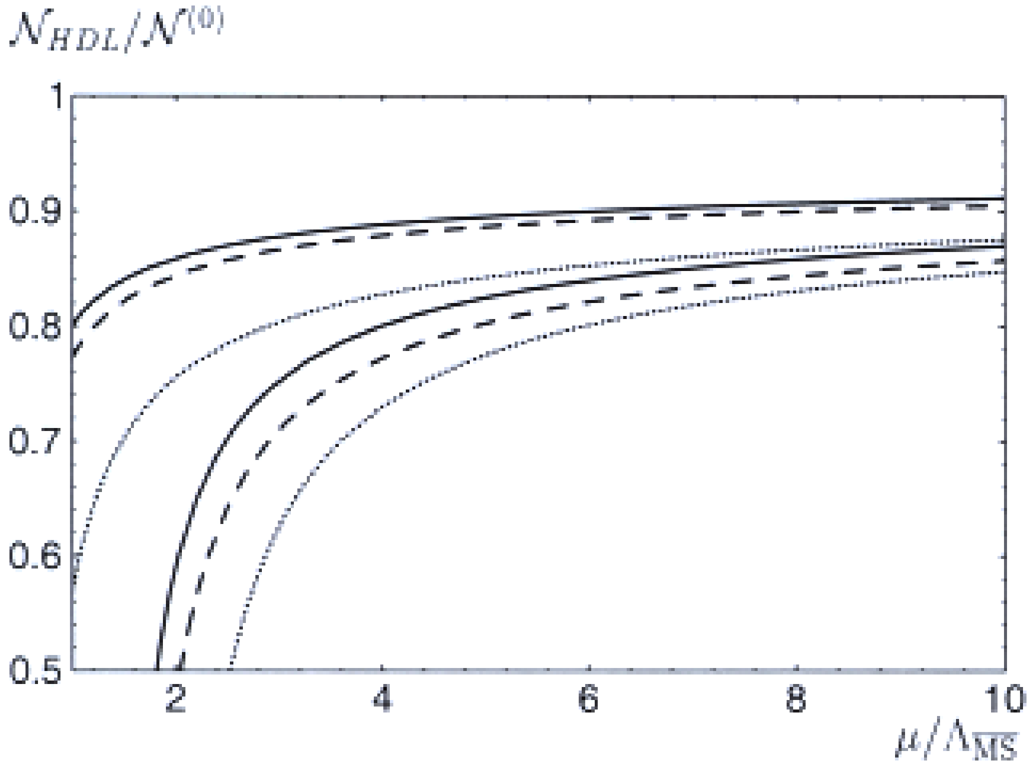
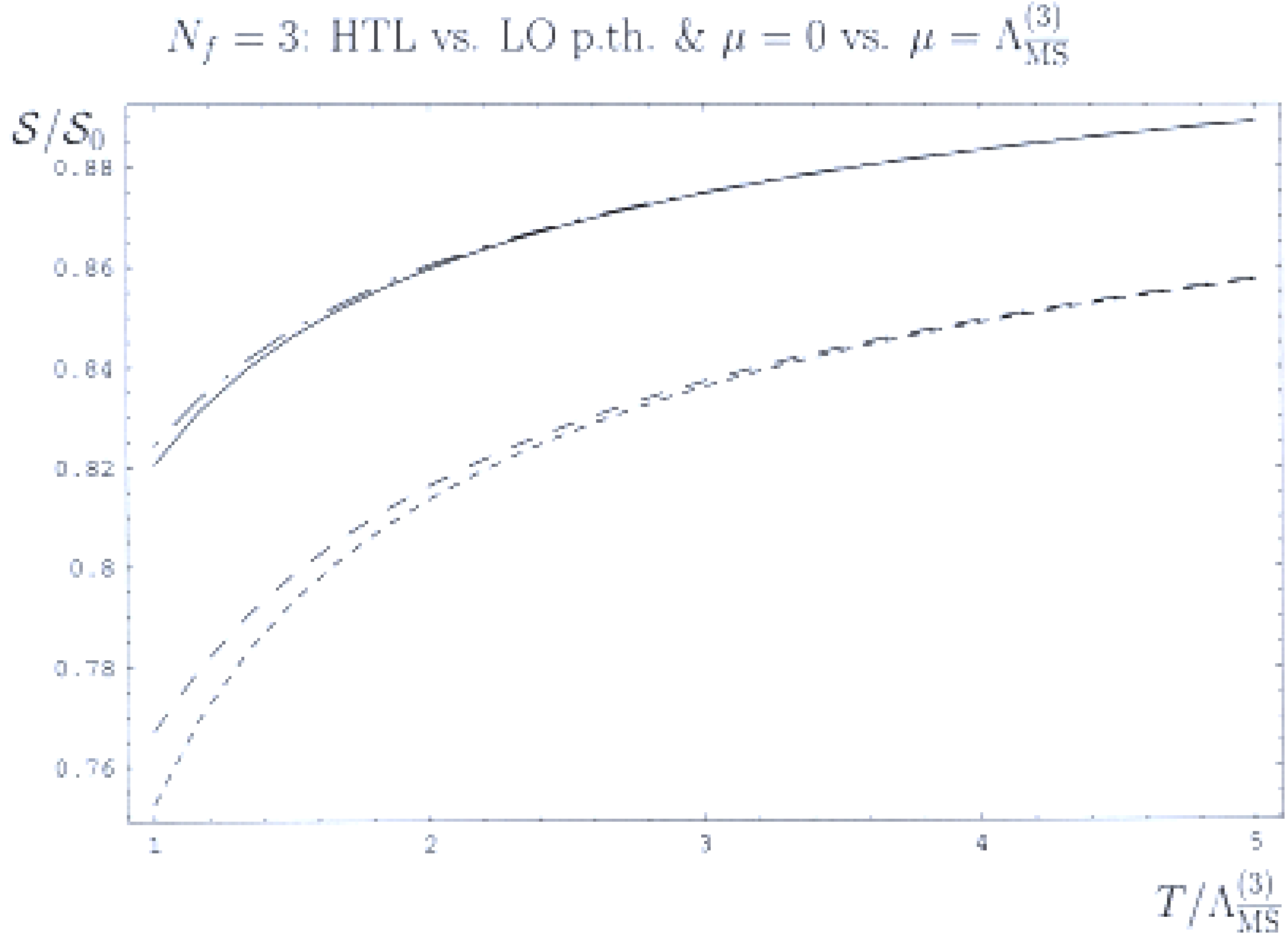


Figure 5: The quark density for $N_f = 3$:
Full lines: HDL-resummed calculation.
Perturbative results at order g^2 (dashed lines) and order g^4 (dotted lines) [Freedman and McLerran (78)]

N.B.: Perturbation theory is much better behaved.

But ... no lattice data !



$N_f = 3$: HTL vs. LO p.th. & $\mu = 0$ vs. $\mu = \Lambda_{\overline{\rm MS}}^{(3)}$
$\mathcal{S}/\mathcal{S}_0$
0.88
0.86
0.84
0.82
0.8
0.78
0.76
1
2
3
4
5
$T/\Lambda_{\overline{\rm MS}}^{(3)}$

CONCLUSIONS

- Good agreement with lattice data when available down to $T \sim 2 - 3T_c$
- Supports the quasiparticle picture of the quark-gluon plasma
- The method makes full use of the spectral information on the quasiparticles
- The approach is easily extended to finite chemical potential
- From $\mathcal{N}(\mu, T)$ and $\mathcal{S}(\mu, T)$ one can reconstruct $\mathcal{P}(\mu, T)$. Use lattice data to fix the integration constant (e.g. $\mathcal{P}(\mu = 0, T)$) [work in progress]