QUARK MATTER 2001

Understanding Color Confinement

A. DI GIACOMO

PISA UNIVERSITY and INFN Sezione di Pisa

Presented by: A. Di Giacomo

Abstract

An updated review is presented of our understanding of color confinement.The implications of Lattice results on condensation of magnetic charges are discussed.The role of vortices is analyzed.

I COLOR CONFINEMENT

BASIC FACTS

I1 CONFINEMENT ≡ ABSENCE OF COLORED PARTICLES IN ASYMPTOTIC STATES

- VERY STRINGENT EXPERIMENTAL LIMITS

$$\frac{n_q}{n_p} \lesssim 10^{-27} \quad \text{vs EXPECT.} \quad \frac{n_q}{n_p} \simeq 10^{-12}$$

⇓

CONFINEMENT IS AN ABSOLUTE PROPERTY TO BE EXPLAINED IN TERMS OF SYMMETRY, LIKE SUPERCONDUCTIVITY

I2 $N_c \to \infty$

QCD IN THE LIMIT $N_c \to \infty$ HAS THE SAME ESSENTIAL PROPERTIES (E.G. CONFINEMENT) AS FOR $N_c = 3$ [t'Hooft, Witten, Veneziano]

$1/N_c$ A GOOD EXPANSION PARAMETER

⇓

MECHANISM OF COLOR CONFINEMENT ~ INDEPENDENT OF N_c

II. DUALITY (KRAMERS + WANNIERS ; KADANOFF ...)

ORDER & DISORDER

- A DEEP CONCEPT IN STATISTICAL MECHANICS AND QUANTUM FIELD THEORY

APPLIES TO SYSTEMS ADMITTING NON LOCAL FIELD CONFIGURATIONS WITH NON-TRIVIAL TOPOLOGY

TWO COMPLEMENTARY DESCRIPTIONS

DIRECT	DUAL
- FIELDS ϕ	- FIELDS μ
- $\langle\phi\rangle$ (ORDER PARAMETERS) IDENTIFY SYMMETRY OF GROUND STATE	- $\langle\mu\rangle$ (DISORDER PARAMETERS) IDENTIFY SYMMETRY OF GROUND STATE
- μ NON LOCAL EXCITATIONS	- ϕ NON LOCAL EXCITATION
- SUITABLE FOR $g \ll 1$ (WEAK COUPLING, OR LOW T)	- SUITABLE FOR $g > 1$ (STRONG COUPLING, OR HIGH T)
	$g_D \sim 1/g$

BY DUALITY THE STRONG COUPLING REGIME OF THE DIRECT DESCRIPTION IS MAPPED IN THE WEAK COUPLING REGIME OF THE DUAL DESCRIPTION. AND VICEVERSA

- ISING MODEL IN 2d (KADANOFF)
- SUSY N=2 QCD (SEIBERG-WITTEN)
- ....

- DUALITY IN QCD

$T = \Lambda \exp\left(\frac{b_0}{g^2}\right)$ (ASYMPTOTIC FREEDOM) $b_0 > 0$

DIRECT	DUAL
- WEAK COUPLING (high T)	STRONG COUPLING (LOW T)
DECONFINED PHASE	CONFINED PHASE
A_μ, ψ, L (POLYAKOV LINE)	$\langle \mu \rangle$ DISORDER PARAMETER
$\langle L \rangle$ = ORDER PARAMETER IN THE ABSENCE OF QUARKS	μ : WEAKLY INTERACTING FIELDS
μ : TO BE IDENTIFIED TOPOLOGICAL EXCITATIONS	$g_D \sim \frac{1}{g} \ll 1$

IF μ WERE KNOWN DESCRIPTION OF CONFINED PHASE WOULD BE PERTURBATIVE IN g_D

STRATEGY

(a) IDENTIFY THE (TOPOLOGICAL) QUANTUM NUMBERS OF μ - SYMMETRY OF CONFINED VACUUM

(b) IDENTIFY THE DUAL EXCITATIONS μ

III TOPOLOGICAL CONFIGURATIONS IN QCD

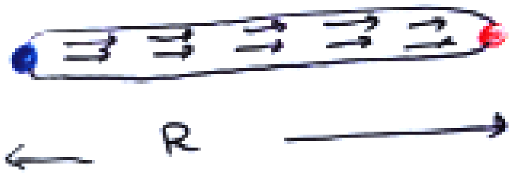
NATURAL TOPOLOGY : MAPPING OF THE SPHERE S_2 AT SPATIAL INFINITY ON A GROUP.

$S_2 \rightarrow SU(2)/Z_2$ $\Longrightarrow$ MONOPOLES

['t HOOFT, POLYAKOV]

- IF μ CARRIES MAGNETIC CHARGE
  $\langle \mu \rangle \neq 0$ SIGNALS CONDENSATION OF MONOPOLES
  OR DUAL SUPERCONDUCTIVITY OF VACUUM
  ['t HOOFT, MANDELSTAM]

- CONFINEMENT BY DUAL-MEISSNER EFFECT



$V \sim \sigma R$

- IDENTIFY MONOPOLES (ABELIAN PROJECTION)
  ('t HOOFT 1981)

$\vec{\Phi}(x)$ A FIELD IN THE ADJOINT REPRESENTATION

$$\hat{\Phi} \equiv \frac{\vec{\Phi}}{|\vec{\Phi}|}$$ [UNDEFINED AT $\vec{\Phi} = 0$]

$$\vec{G}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g \vec{A}_\mu \wedge \vec{A}_\nu$$ (FIELD STRENGTH TENSOR)

$$D_\mu = \partial_\mu + g \vec{A}_\mu \wedge$$

$$F_{\mu\nu} = \hat{\Phi} \cdot \vec{G}_{\mu\nu} - \frac{1}{g} \hat{\Phi} (D_\mu \hat{\Phi} \wedge D_\nu \hat{\Phi})$$

$$= \hat{\Phi} (\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu) - \frac{1}{g} \hat{\Phi} (\partial_\mu \hat{\Phi} \wedge \partial_\nu \hat{\Phi})$$

$$= \partial_\mu (\hat{\Phi} \vec{A}_\nu) - \partial_\nu (\hat{\Phi} \vec{A}_\mu)$$ IN THE GAUGE $\hat{\Phi} = \overrightarrow{\text{const}}$ e.g. $\hat{\Phi} = (0, 0, 1)$

$$F^*_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} \qquad j^M_\nu = \partial_\mu F^*_{\mu\nu}$$ (MAGNETIC CURRENT)

$$\boxed{\partial^\nu j^M_\nu = 0}$$

A CONSERVED MAGNETIC CHARGE FOR ANY CHOICE OF $\vec{\Phi}$!! AN INFINITY OF THEM!!

IV DETECTING THE SYMMETRY OF THE CONFINING VACUUM.

- CONSTRUCT A NON LOCAL OPERATOR μ_a CREATING MAGNETIC MONOPOLES IN A GIVEN ABELIAN PROJECTION [A. Di Giacomo et al 1995 → 2000]

- MEASURE $\langle \mu_a \rangle$ TO DETECT CONDENSATION OF MAGNETIC CHARGES (DUAL SUPERCONDUCTIVITY)

A TYPICAL BEHAVIOUR OF $\langle \mu_a \rangle$ (FIG. I)

FOR TECHNICAL REASONS THE MEASUREMENT OF

$$\rho = \frac{d}{d\beta} \ln \langle \mu_a \rangle \qquad \beta = \frac{2N_c}{g^2}$$

IS PREFERRED (FIG II, III)

A SPECTACULAR NEGATIVE PEAK AT THE PHASE TRANSITION

- A FINITE SIZE SCALING ANALYSIS GIVES THE LIMIT $V \to \infty$ AND THE CRITICAL INDICES [FIG IV, V]

- ρ DOES NOT DEPEND ON THE CHOICE OF THE ABELIAN PROJECTION

EVEN WITH NO GAUGE FIXING THE BEHAVIOUR IS THE SAME

- THE PRESENCE OF QUARKS DOES NOT MODIFY THE PICTURE [Fig VI, Fig VII]

BELOW THE CHIRAL TRANSITION THERE IS DUAL SUPERCONDUCTIVITY, INDEPENDENT OF THE ABELIAN PROJECTION

WORK IN PROGRESS TO DETERMINE THE CRITICAL INDICES.

REMARKS:

(i) WHATEVER THE DUAL FIELDS, THEY MUST HAVE NON-ZERO MAGNETIC CHARGE IN ALL THE ABELIAN PROJECTIONS (MONOPOLES OF THE MAXIMAL ABELIAN GAUGE DO NOT)

(ii) THE PRESENCE OF FERMIONS DOES NOT MODIFY THE PATTERN. $N_c \to \infty$ IS OK

(iii) WHO ARE THE FUNDAMENTAL DUAL EXCITATIONS? PROBLEM IS OPEN FOR COMPETITION.

V VORTICES ('t HOOFT 78)

STUDY OF VORTICES (Z_{N_c}) CAN PROVIDE HINTS TO SOLVE (iii)

- VORTICES ARE STRINGLIKE TOPOLOGICAL DEFECTS ON A LINE C

$$B(C)\, W(C') = W(C')\, B(C)\, e^{i \frac{n_{CC'}}{N} 2\pi}$$

('t HOOFT, POLYAKOV)

$W(C')$ WILSON LOOP ; $n_{CC'}$ WINDING # OF C, C'

- THEOREM: IF W OBEYS THE AREA LAW ($\sigma \neq 0$) THEN B OBEYS THE PERIMETER LAW ($\sigma' = 0$) AND VICEVERSA ('t HOOFT)

B(C) CAN BE CONSTRUCTED ON A LATTICE. $\langle B(C) \rangle$ IS MEASURED AND THEOREM CAN BE CHECKED

- CHOOSE FOR C A LINE ALONG z AXIS FROM $-\infty$ TO $+\infty$ (DUAL POLYAKOV LINE, C_p) (L. Del Debbio, A. Di Giacomo, B. Lucini N.P.B 2001; Phys Lett B to appear)

(iv) $\langle B(C_p) \rangle$: • fig VII $\quad \frac{d}{d\beta} \ln \langle B(C_p) \rangle - \rho_V$

• FINITE SIZE SCALING FIG VIII

SAME BEHAVIOUR AS $\langle \mu_a \rangle$

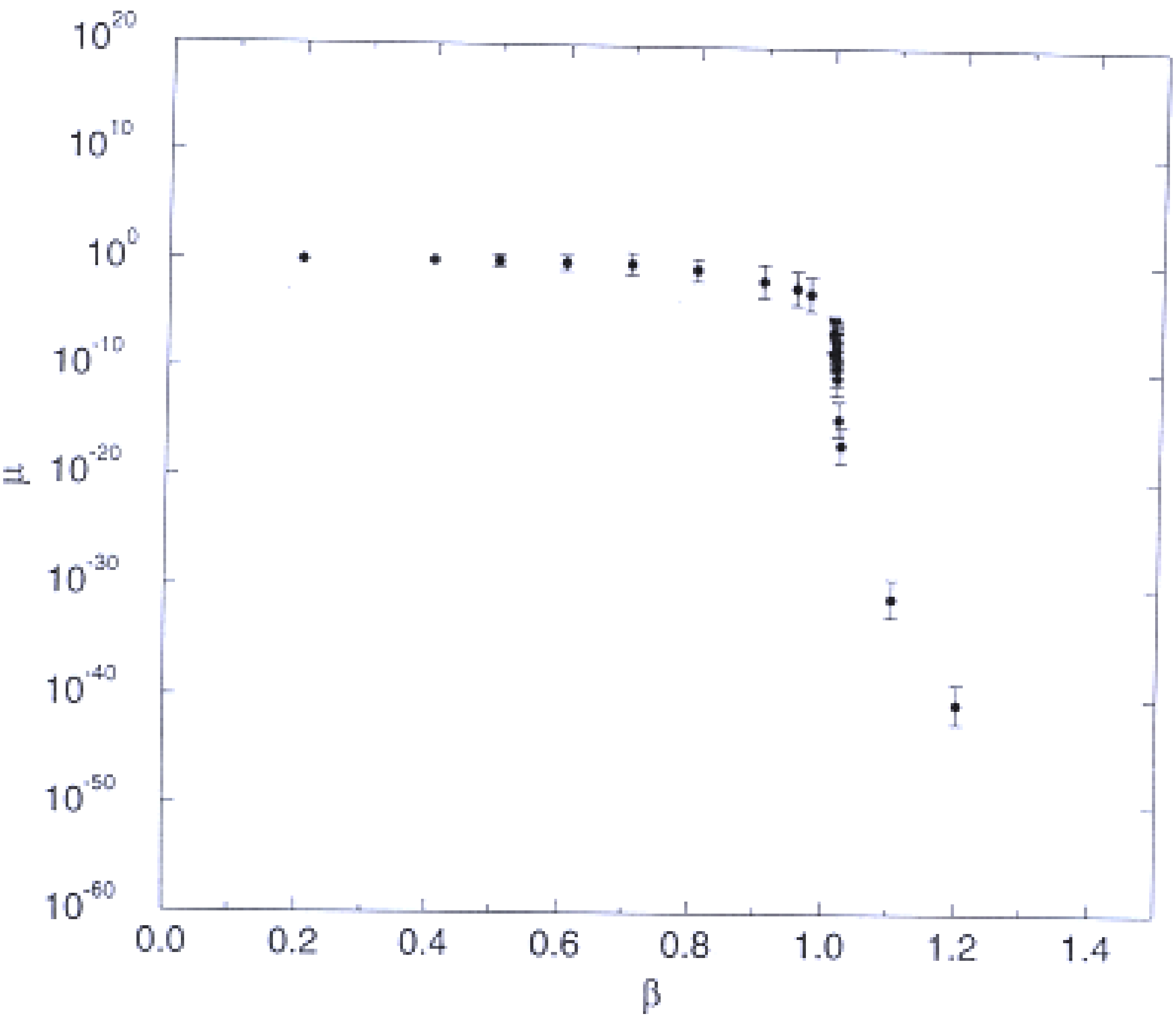
CONCLUSIONS

- CONFINEMENT RELATED TO SYMMETRY: DUALITY IS AT WORK
- CONFINING VACUUM IS A DUAL SUPERCONDUCTOR IN ALL ABELIAN PROJECTION
  FUNDAMENTAL DUAL FIELDS HAVE MAGNETIC CHARGE IN ALL ABELIAN PROJECTIONS.
- 'tHOOFT ALGEBRA FOR DUAL VORTICES CHECKED.
  DUAL POLYAKOV LINE IS A GOOD DISORDER PARAMETER.

- OPEN PROBLEM

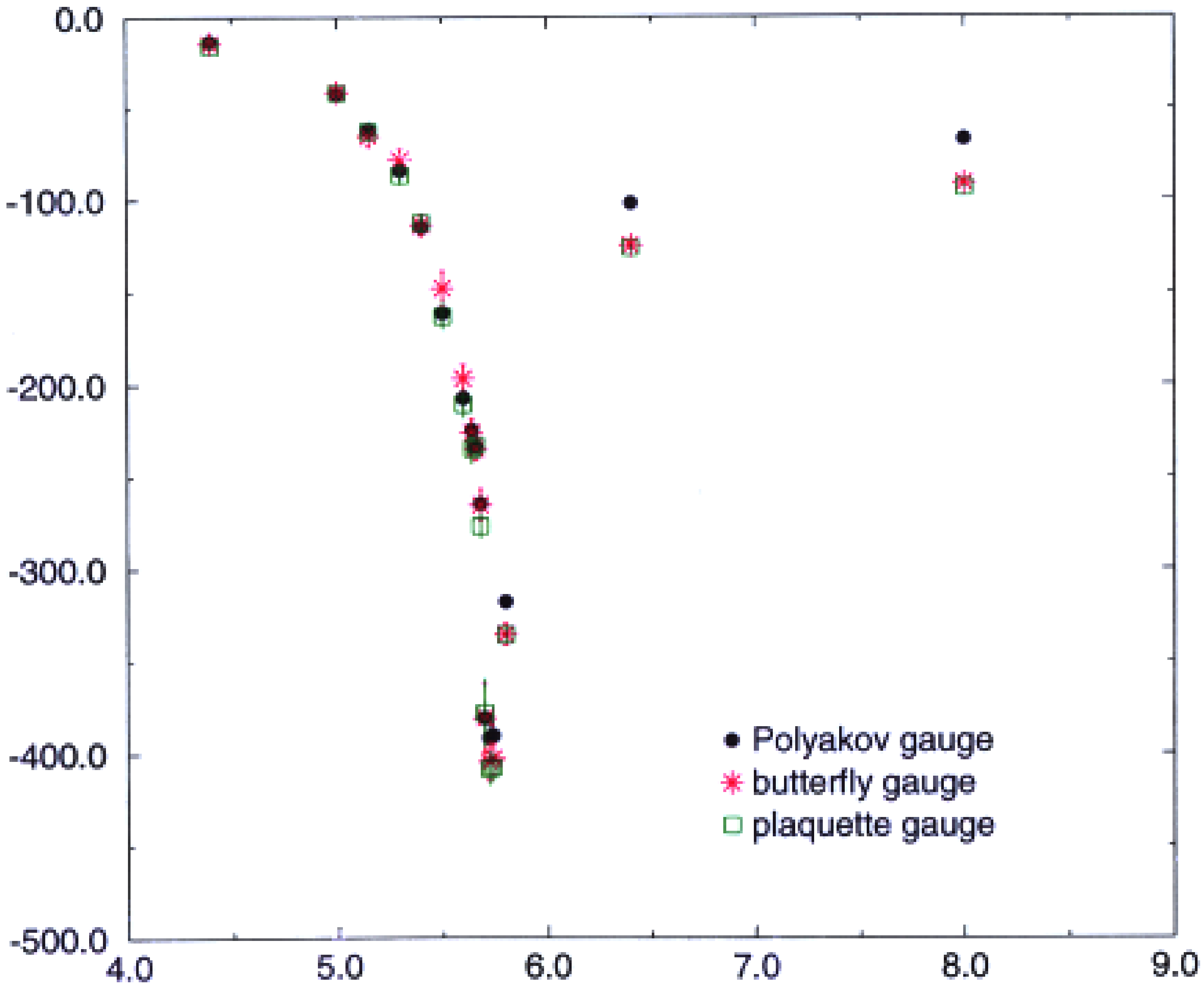
  IDENTIFY DUAL FIELDS + EFFECTIVE LAGRANGEAN

  DUAL EXCITATIONS COULD BE DETECTABLE AT THE PHASE TRANSITION



10^{20}
10^{10}
10^{0}
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}
10^{-60}
μ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
β

SU(3)

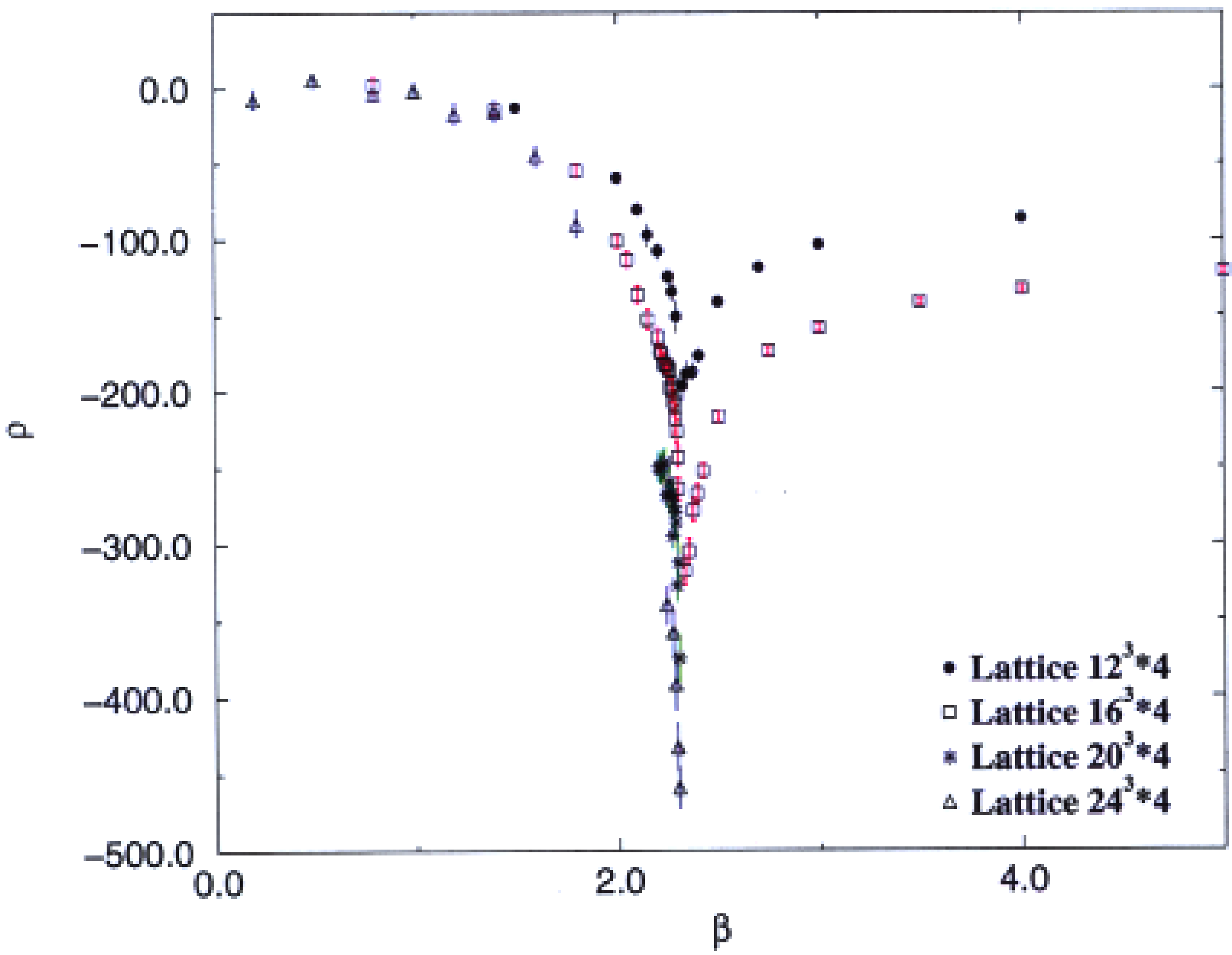


0.0
-100.0
-200.0
-300.0
-400.0
-500.0
4.0
5.0
6.0
7.0
8.0
9.0
Polyakov gauge
butterfly gauge
plaquette gauge

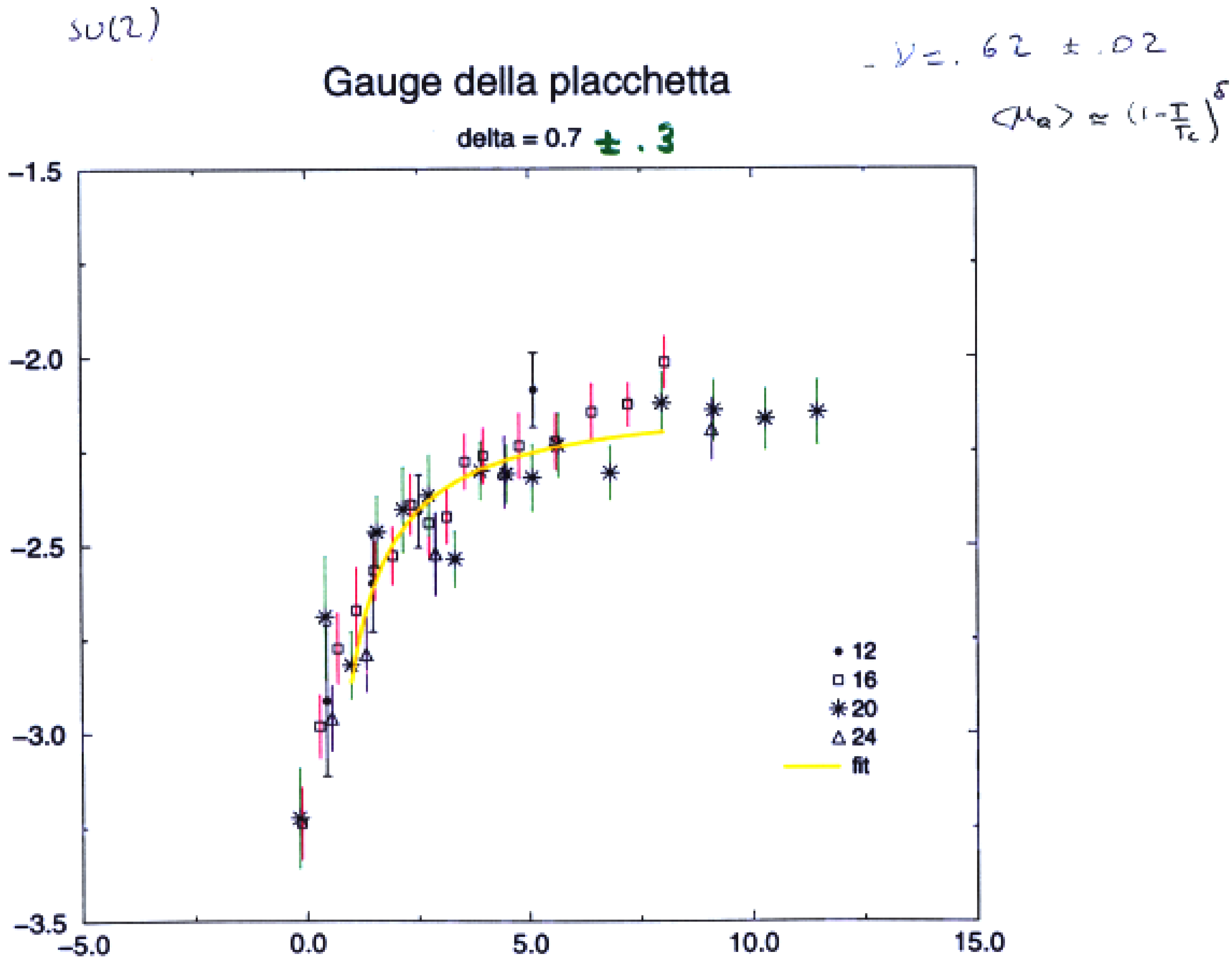
$\beta > \beta_c$

$\rho \sim \exp(-c N_s)$

SU(2) Lattice Gauge Theroy
Plaquette Gauge



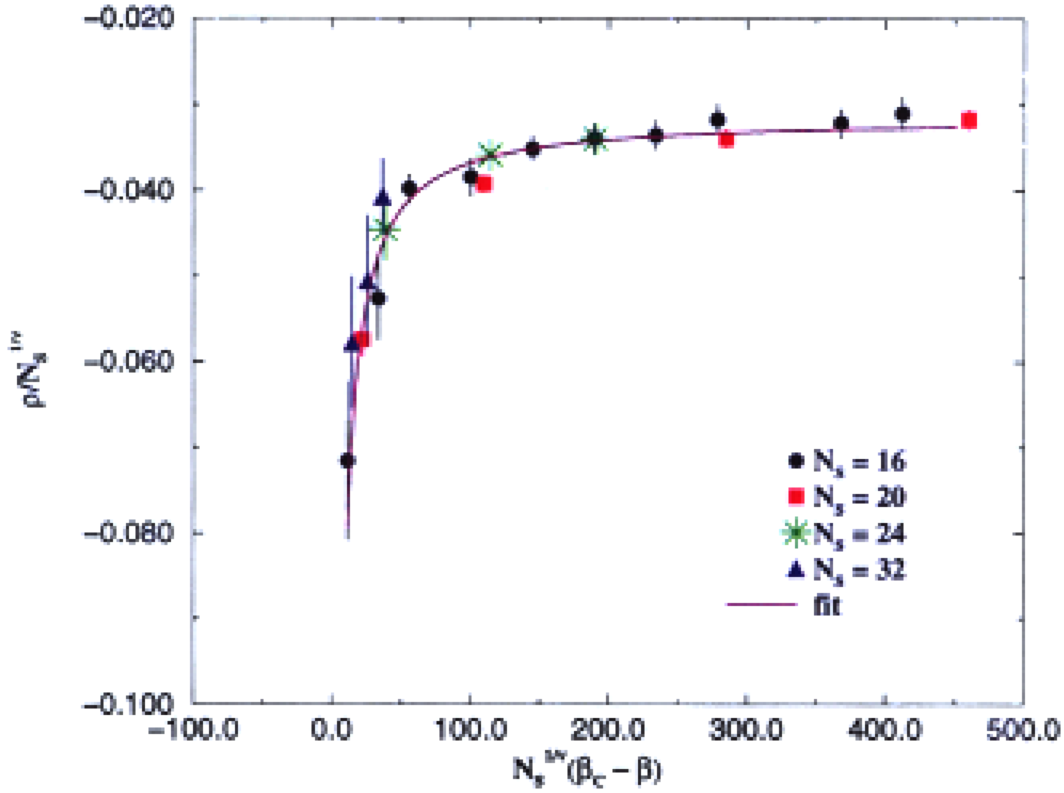
3



SU(2)
Gauge della placchetta
delta = 0.7 ± .3
$-\nu = .62 \pm .02$
$\langle \mu_e \rangle \simeq (1 - \frac{T}{T_c})^\delta$
−1.5
−2.0
−2.5
−3.0
−3.5
−5.0
0.0
5.0
10.0
15.0
12
16
20
24
fit

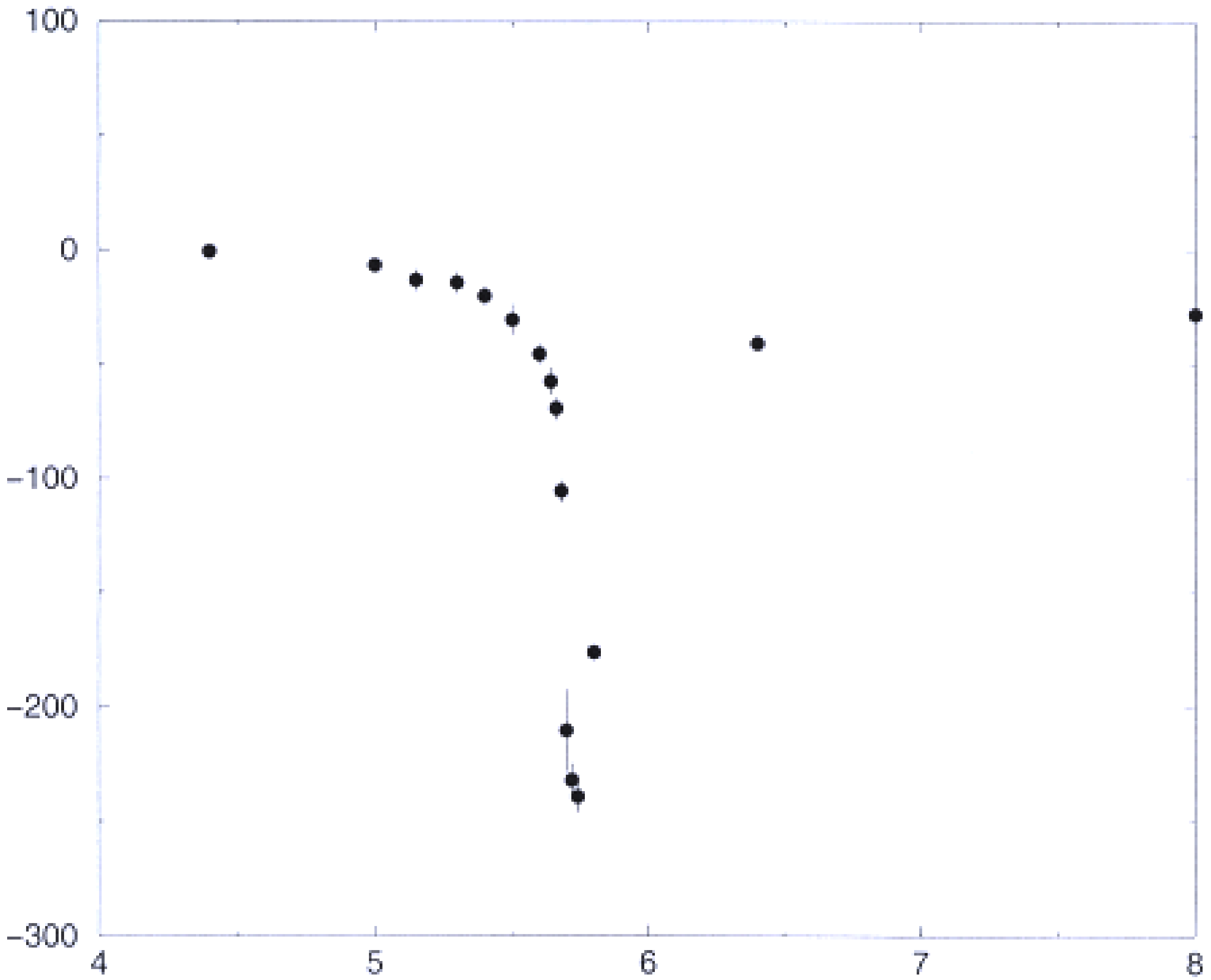
4

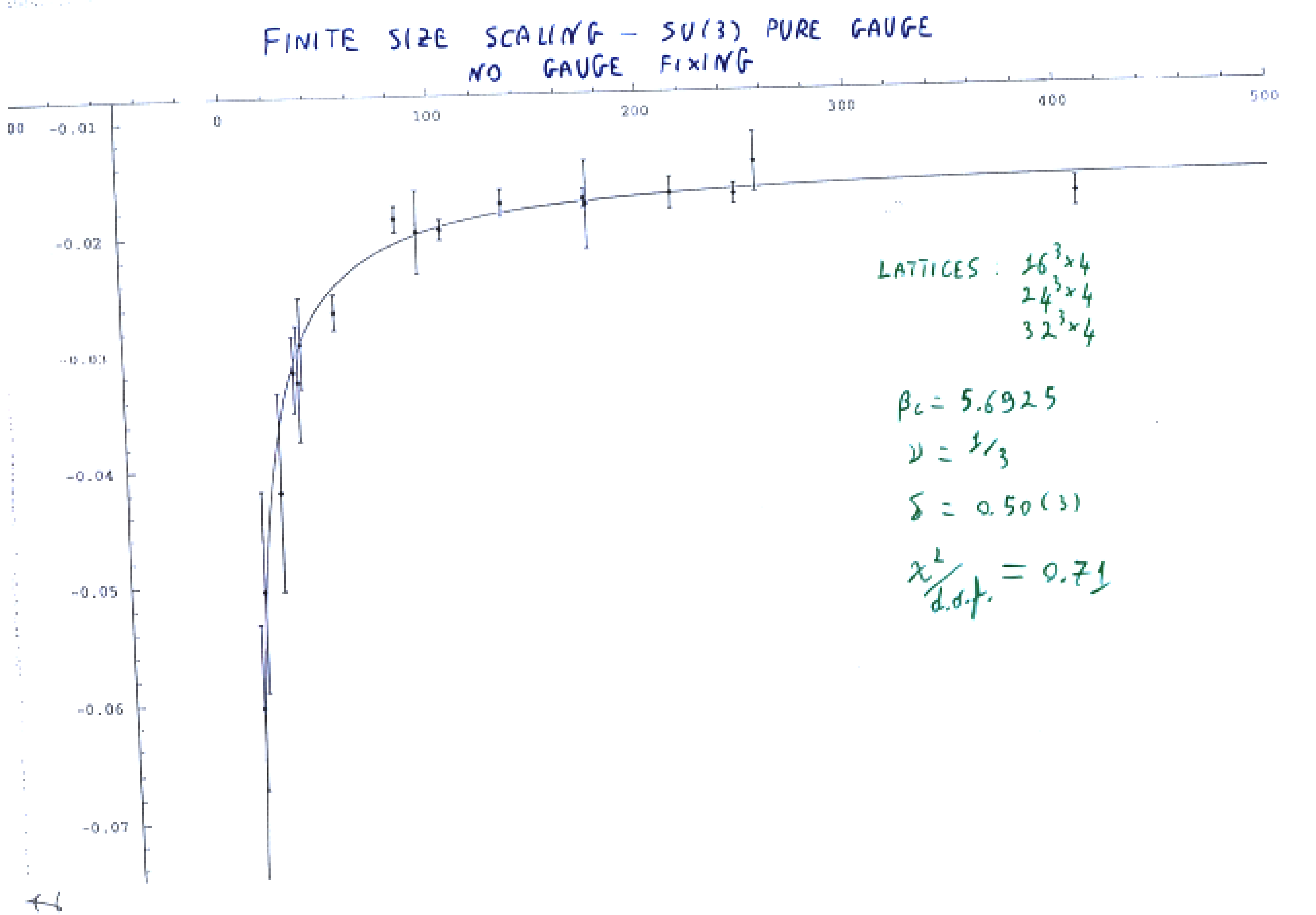
Critical region
SU(3) LGT, Polyakov projection

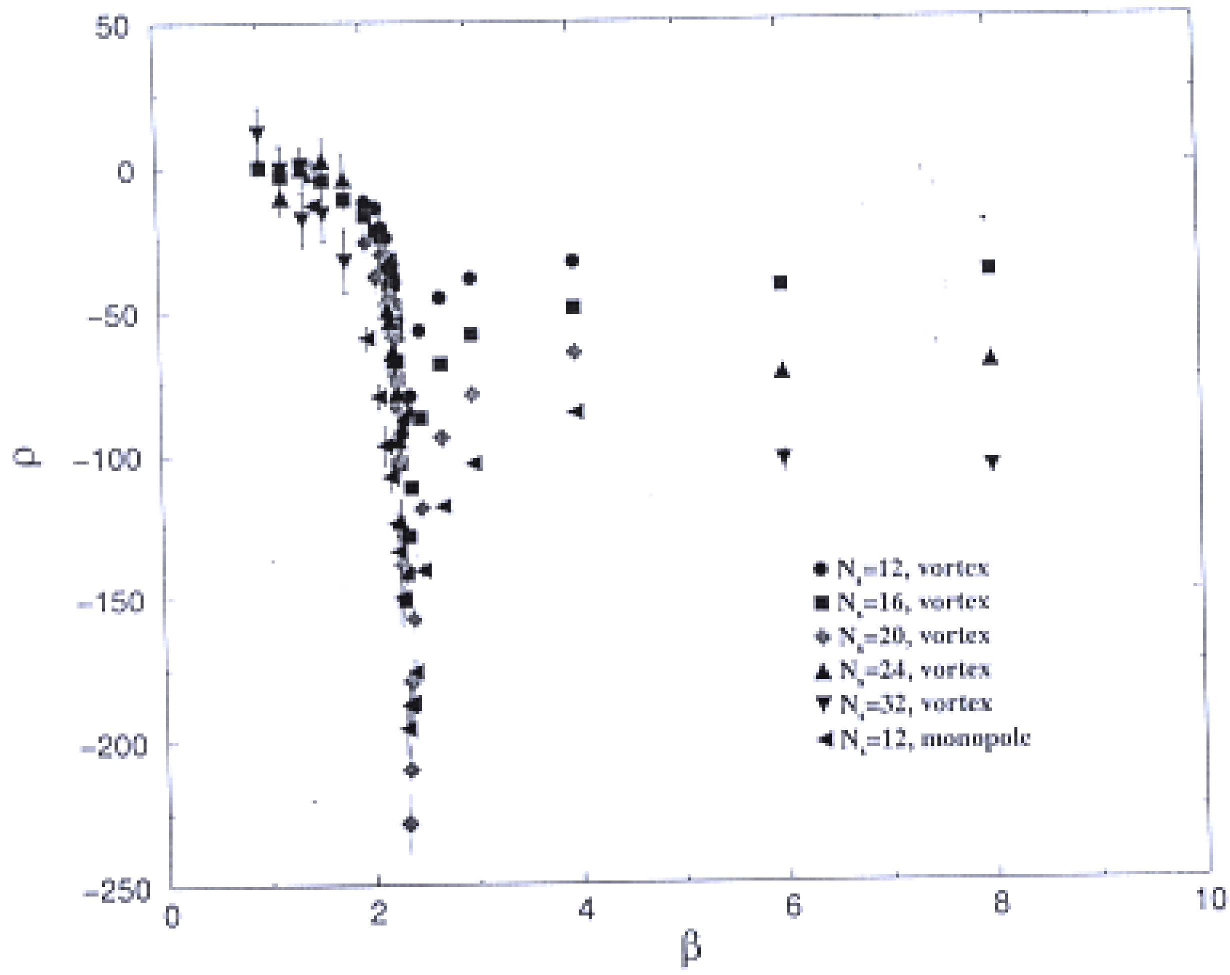


Fit result: $\delta = 0.54(4)$

ν= .33

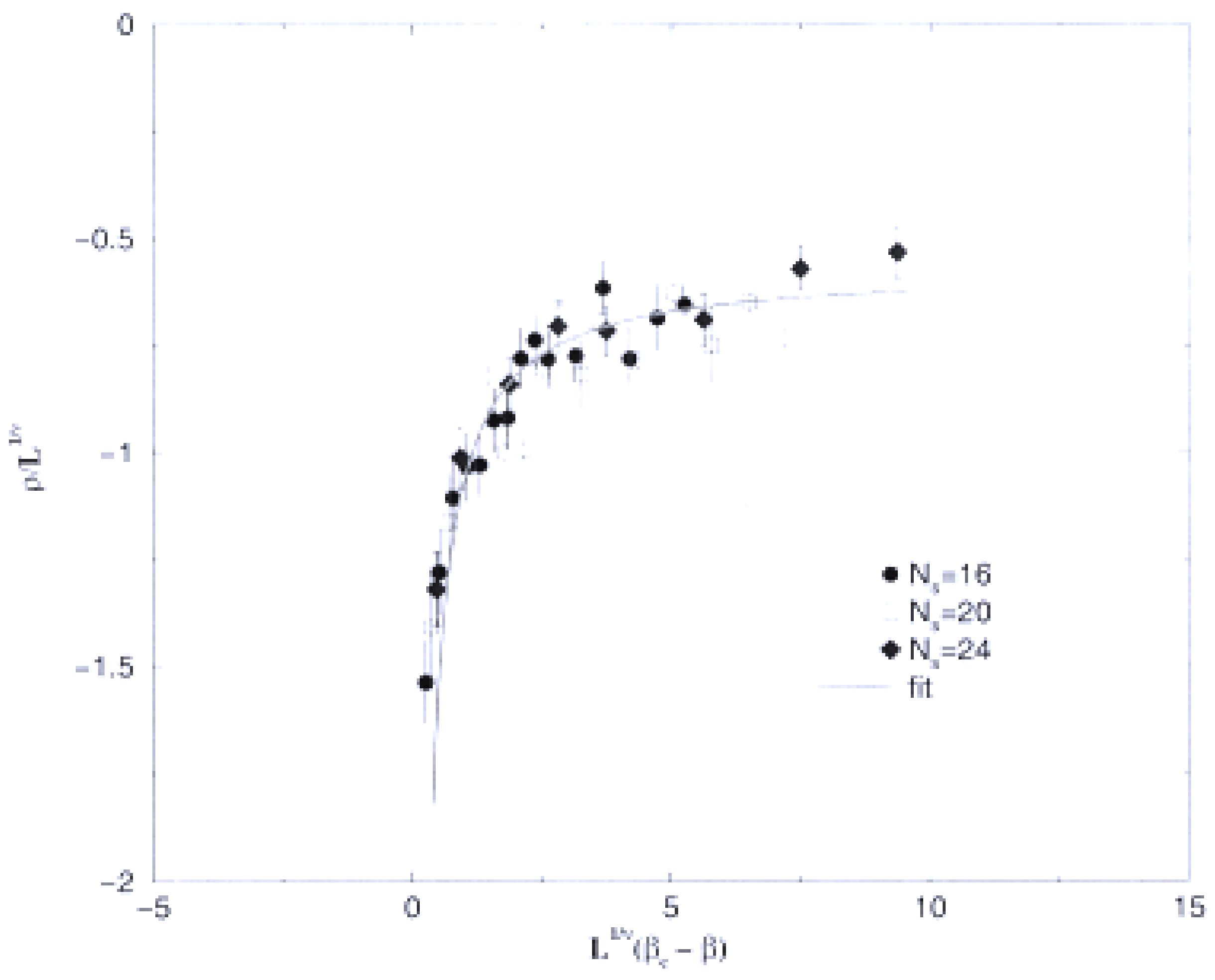

100
0
−100
−200
−300
4
5
6
7
8


FINITE SIZE SCALING – SU(3) PURE GAUGE
NO GAUGE FIXING
0
100
200
300
400
500
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
LATTICES : 16³×4
24³×4
32³×4
βc = 5.6925
ν = 1/3
δ = 0.50(3)
χ²/d.o.f. = 0.71



50
0
-50
-100
-150
-200
-250
ρ
0
2
4
6
8
10
β
N_t=12, vortex
N_t=16, vortex
N_t=20, vortex
N_t=24, vortex
N_t=32, vortex
N_t=12, monopole

Fig. 7


0
-0.5
-1
-1.5
-2
-5
0
5
10
15
N_s=16
N_s=20
N_s=24
fit