Understanding Color Confinement

A. DI GIACOMO

PISA UNIVERSITY and INFN Sezione di Pisa

Presented by: A. Di Giacomo

Abstract

An updated review is presented of our understanding of color confinement. The implications of Lattice results on condensation of magnetic charges are discussed. The role of vortexes is analyzed.
I COLOR CONFINEMENT


II CONFINEMENT ≡ ABSENCE OF COLORED PARTICLES IN ASYMPTOTIC STATES

- VERY STRINGENT EXPERIMENTAL LIMITS

\[ \frac{n_q}{n_p} \approx 10^{-27} \quad \text{versus expectation} \quad \frac{n_q}{n_p} \approx 10^{-12} \]

\[ \downarrow \]

CONFINEMENT IS AN ABSOLUTE PROPERTY TO BE EXPLAINED IN TERMS OF SYMMETRY, LIKE SUPERCONDUCTIVITY


III \( N_c \to \infty \)

QCD IN THE LIMIT \( N_c \to \infty \) HAS THE SAME ESSENTIAL PROPERTIES (E.G. CONFINEMENT).

AS FOR \( N_c = 3 \) \( [\text{t'Hooft, Witten, Veneziano}] \)

\[ \frac{N_c}{N_c} \quad \text{a good expansion parameter} \]

\[ \downarrow \]

MECHANISM OF COLOR CONFINEMENT ∼ INDEPENDENT OF \( N_c \).
Duality (Kramers-Wanniers; Kadanoff...)

**Order & Disorder**

A deep concept in statistical mechanics and quantum field theory.

Applies to systems admitting non-local field configurations with non-trivial topology.

Two complementary descriptions.

**Direct**

- Fields $\phi$
  - $\langle \phi \rangle$ (Order parameters)
  - Identifies symmetry of ground state
  - $\phi$ non-local excitations
  - Suitable for $g \ll 1$
    (Weak coupling, or low $T$)

**Dual**

- Fields $\mu$
  - $\langle \mu \rangle$ (Disorder parameters)
  - Identifies symmetry of ground state
  - $\phi$ non-local excitation
  - Suitable for $g > 1$
    (Strong coupling, or high $T$)
  - $q_d \sim 1/g$

By duality the strong coupling regime of the direct description is mapped in the weak coupling regime of the dual description, and vice versa.

Ising model in 2d (Kadanoff)

SUSY N=2 QCD

(Seiberg-Witten)
DUALITY IN QCD

\[ T = A \exp \left( \frac{b_0}{g^2} \right) \] (ASYMPTOTIC FREEDOM)

**DIRECT**

- WEAK COUPLING (HIGH T)
- DECONFINED PHASE
- \( A_\mu , \psi , \xi \) (POLYAKOV LINES)
- \( \langle L \rangle = \) ORDER PARAMETER IN THE ABSENCE OF QUARKS
- \( \mu : \) TO BE IDENTIFIED TOPOLOGICAL EXCITATIONS

**DUAL**

- STRONG COUPLING (LOW T)
- CONFINED PHASE
- \( \mu : \) DISORDER PARAMETER
- \( \mu : \) WEAKLY INTERACTING FIELDS
- \( g_0 \sim \frac{1}{g} \ll 1 \)

IF \( \mu \) WERE KNOWN, DESCRIPTION OF CONFINED PHASE WOULD BE PERTURBATIVE IN \( g_0 \)

**STRATEGY**

(a) IDENTIFY THE (TOPOLOGICAL) QUANTUM NUMBERS OF \( \mu \) - SYMMETRY OF CONFINED VACUUM

(b) IDENTIFY THE DUAL EXCITATIONS \( \mu \)
III. Topological Configurations in QCD

Natural Topology: Mapping of the sphere $S^2$ at Spatial Infinity on a Group.

$S^2 \rightarrow SU(2)/Z_2 \rightarrow \text{Monopoles}$

- If $\mu$ Carries Magnetic Charge

$\langle \mu \rangle \neq 0$ signals condensation of monopoles or dual superconductivity of vacuum

[Te 't Hooft, Polyakov]

- Confinement by Dual-Heissner Effect

\[ B = \alpha \rightarrow \gamma \Rightarrow \nu \sim \sigma \text{R} \leftarrow \gamma \rightarrow \text{R} \rightarrow \]

- Identify Monopoles (Abelian Projection)

(['e 't Hooft 1981]

$\tilde{\phi}$ is a field in the adjoint representation

$\hat{\phi} \equiv \frac{\tilde{\phi}}{|\tilde{\phi}|}$ (Undefined at $\tilde{\phi} = 0$)

$G_{\mu \nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu + g \tilde{A}_\mu \wedge \tilde{A}_\nu$ (Field Strength Tensor)

$D_\mu = \partial_\mu + g \tilde{A}_\mu$

$F_{\mu \nu} = \hat{\phi} \cdot G_{\mu \nu} - \frac{1}{2} \hat{\phi} (D_\mu \hat{\phi} \wedge D_\nu \hat{\phi})$

$= \hat{\phi} (\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu) - \frac{1}{2} \hat{\phi} (\partial_\mu \hat{\phi} \wedge \partial_\nu \hat{\phi})$

$= \partial_\mu (\hat{\phi} \tilde{A}_\nu) - \partial_\nu (\hat{\phi} \tilde{A}_\mu)$ in the gauge $\hat{\phi} = \text{const}$

e.g. $\hat{\phi}(0,0,1)$

$F^\nu_{\mu} = \frac{1}{2} \varepsilon_{\mu \nu \alpha \beta} F^{\alpha \beta} \Rightarrow j^\nu = \partial_\nu F^{\mu \nu}$ (Magnetic Current)

$\partial^\nu j^\mu = 0$ A conserved magnetic charge

For any choice of $\phi$!! An infinity of them!!
DETECTING THE SYMMETRY OF THE CONFINING VACUUM.

- CONSTRUCT A NON LOCAL OPERATOR \( \mu_\alpha \) CREATING MAGNETIC MONPOLES IN A GIVEN ABELIAN PROJECTION [A. Di Giorgio et al. 1995, 2000]

- MEASURE \( \langle \mu_\alpha \rangle \) TO DETECT CONDENSATION OF MAGNETIC CHARGES (DUAL SUPERCONDUCTIVITY)

A TYPICAL BEHAVIOUR OF \( \langle \mu_\alpha \rangle \) (FIG. I)

FOR TECHNICAL REASONS THE MEASUREMENT OF

\[
\beta \equiv \frac{d}{d\beta} \ln \langle \mu_\alpha \rangle \quad \beta = \frac{2N_c}{g^2}
\]

IS PREFERRED (FIG. II, III)

A SPECTACULAR NEGATIVE PEAK AT THE PHASE TRANSITION

A FINITE SIZE SCALING ANALYSIS GIVES THE LIMIT \( V \to \infty \) AND THE CRITICAL INDICES (FIG. IV, V)

\( g \) DOES NOT DEPEND ON THE CHOICE OF THE ABELIAN PROJECTION,

EVEN WITH NO GAUGE FIXING THE BEHAVIOUR IS THE SAME

- THE PRESENCE OF QUARKS DOES NOT MODIFY THE PICTURE (FIG. VI, FIG. VII)

BELOW THE CHIRAL TRANSITION THERE IS DUAL SUPERCONDUCTIVITY, INDEPENDENT OF THE ABELIAN PROJECTION

WORK IN PROGRESS TO DETERMINE THE CRITICAL INDICES.
REMARKS:

(i) Whatever the dual fields, they must have non-zero magnetic charge in all the Abelian projections (monopoles of the maximal Abelian gauge do not)

(ii) The presence of fermions does not modify the pattern. $N_c \to \infty$ is ok

(iii) Who are the fundamental dual excitations? Problem is open for competition.

V ORTICES [’tHooft ’78]

Study of vortices ($Z_{N_c}$) can provide hints to solve (iii)

- Vortices are stringlike topological defects on a line $c$

$$B(c) W(c') = W(c') B(c) e^{i \frac{2 \pi}{N} c c'} \quad \text{[’tHooft, Polyakov]}$$

$W(c')$ Wilson loop; $w_{cc'}$ winding # of $c, c'$

- Theorem: If $W$ obeys the area law ($\sigma \neq 0$) then $B$ obeys the perimeter law ($\sigma' = 0$) and vice versa (’tHooft)

$B(c)$ can be constructed on a lattice, $\langle B(c) \rangle$ is measured and Theorem can be checked

- Choose for $c$ a line along $\beta$ axis from $-\infty$ to $\infty$

$$\langle B(Cp) \rangle = \frac{1}{N} \frac{\partial}{\partial \beta} \langle B(Cp) \rangle \quad \text{finite size scaling Fig VIII}$$

Same behavior as $\langle \mu_0 \rangle$
CONCLUSIONS

- Confineent related to symmetry: duality is at work.

- Confining vacuum is a dual superconductor in all abelian projection fundamental dual fields have magnetic charge in all abelian projections.

- 'tHooft algebra for dual vortices checked. Dual Polyakov line is a good disorder parameter.

OPEN PROBLEM

Identify dual fields & effective Lagrangean.

Dual excitations could be detectable at the phase transition.
\[ \beta > \beta_c \]
\[ g \sim \exp(-cN_s) \]

SU(2) Lattice Gauge Theory
Plaquette Gauge

- Lattice $12^3\times4$
- Lattice $16^3\times4$
- Lattice $20^3\times4$
- Lattice $24^3\times4$
Gauge della placchetta

\[ \delta = 0.7 \pm 0.3 \]

\[ \langle \mu_0 \rangle \approx (1 - \frac{T}{T_c})^\delta \]

su(2)
Critical region
SU(3) LGT, Polyakov projection

Fit result: $\delta = 0.54(4)$
FINITE SIZE SCALING - SU(3) PURE GAUGE
NO GAUGE FIXING

LATTICES: 16^3x4
          24^3x4
          32^3x4

\beta_c = 5.6925
\nu = 1/3
\delta = 0.50 (3)

\chi^{2}_{d.o.f.} = 0.71